



## Optimization of imprecise redundancy allocation problems for a complicated system using soft computing technique

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### Abstract

Component's selection and perfect level of redundancy selection for maximizing system reliability is main purpose of the redundancy allocation problem (RAP). In this paper we are interested to solve a complex bridge system reliability under some constraints of design. The RAP is designed in crisp environment as well as in imprecise environments to clarify the uncertainty of the model. The decision parameters are made imprecise using triangular fuzzy as well as triangular intuitionistic fuzzy numbers. Graded mean integration approach of crispification is used to crispify the different parameters of constrained fuzzy and intuitionistic fuzzy optimization problems. The constrained optimization problem is converted into unconstrained one using Big-M penalty approach. An advanced genetic algorithm (GA) is applied to solve the reliability optimization problem in precise and imprecise environments. At the end of this study a numerical example is solved and the outcomes are analyzed graphically with respect to the GA parameters.

**Keywords:** Redundancy allocation problem (RAP), Advanced genetic algorithm, Triangular fuzzy number (TFN), Triangular intuitionistic fuzzy number (TIFN), Graded mean integration value (GMIV).

### 1. Introduction

The probability of successful operation of a device or a system in a specified life time under some predetermined restrictions is called the reliability of that system. Reliability optimization is an important branch of advanced operations research. The system reliability can be enhanced in several ways among them two important ways are (i) increment of component reliability and (ii) keeping redundant units parallelly in each subsystem. In case of parallel redundancy, the optimization problem is known as redundancy allocation problem (RAP). The costs, volumes, components' reliabilities, weights etc. are known to the

researchers in RAP type of problems. The main purpose RAP type of problems is to maximize the system reliability under several restrictions by finding optimum number of redundant components, situated actively in each subsystem.

In this paper we have taken redundancy allocation problems due to its NP-hard nature. Kuo and Prasad provided an overview on reliability optimization problem [15]. An integer programming problem in association with a reliable system was solved by Mishra and Sharma [20]. Tillman et al. solved a redundancy allocation problem in their famous article [27]. The problems which are very hard to solve on employing the existing optimization techniques is referred as NP-hard problems [2,8]. Mahapatra and company employed an optimization technique to solve a production-inventory system [17]. Various types of reliability optimization problems are documented in the literature [1,4]. Mahato et al. and Sahoo et al. computed the system reliability of their considered reliability optimization models under different uncertain atmospheres [18,23-24]. Differently designed reliability systems were optimized by Tillman et al. and Tzafestas et al. in their respective studies [28, 29]. Paramanik et al. solved a complicated system using heuristic algorithm for optimizing system reliability in imprecise environment [32].

Reliability optimization problem deals with non-linear objective function. These problems are of the type integer or mixed integer or combination of integer and mixed integer. As a result, heuristic and evolutionary algorithms work efficiently for solving these types of optimization problems. These algorithms do not depend on the continuity or discreteness of the searching space. Various types of deterministic approach such as heuristic methods [16,22], reduced gradient technique [10,11], surrogate-constraints algorithm [9,21], branch and bound method [14,26], dynamic programming method, linear programming approach were employed to obtain the optimum of differently designed reliable systems. Furthermore, differently coded evolutionary algorithms [3,5] and its' modified forms [12-13,25] were used to solve

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redundancy allocation problems under several constraints.

Most of the researches on optimization problems tried to solve the reliability optimization problems with respect to the fixed values of the control parameters. But it is not always good to consider the control parameters as crisp valued due the diversity nature of the problems. To get rid of this situation we can consider the control parameters as uncertain numbers. Very few works are done on imprecise reliability optimization models in the literature. However, some of the researchers employed interval, stochastic, fuzzy, stochastic-fuzzy numbers to optimize reliable systems in imprecise environments. The generalization of crisp number is done by fuzzy number whereas the generalization of fuzzy number is done by intuitionistic fuzzy number. The fuzzy numbers deal with membership value only. But intuitionistic fuzzy numbers deal with both membership and non-membership functions. As a result, the problem becomes more robust in intuitionistic fuzzy environment. Thus, to investigate the optimality of the reliable system impreciseness can be done using intuitionistic fuzzy numbers. According to our knowledge, very few of them tried to solve the reliability optimization using intuitionistic fuzzy atmospheres. Seikh et al. (2012) explain the generalization of triangular fuzzy numbers in intuitionistic fuzzy environment [33]. In the next year they included a note on triangular intuitionistic fuzzy environment [34]. The researchers employed IWO, PSO, QPSO, ABC, GA [6] etc. as evolutionary and heuristic algorithms to handle the imprecise reliability optimization problems. As a result, we are interested to implement fuzzy as well as intuitionistic fuzzy numbers to frame the optimization problems using newly coded evolutionary algorithm.

In this paper, we have considered mixed integer non-linear programming problems in which component reliabilities of each subsystem are crisp, fuzzy and intuitionistic fuzzy valued respectively. Graded mean integration technique [30] is applied to obtain the corresponding crispified models. After that an advanced GA in combination of Big-M penalty technique is used to tackle our proposed problems.

The remainder of this article is arranged as follows: Section 2 and its subsections contain some preliminary definitions of fuzzy and intuitionistic fuzzy numbers and its' defuzzification techniques. Section 3 included the needed assumptions and notation that have been utilized throughout the paper by creating a separate subsection. The RAP is formulated in crisp, fuzzy and intuitionistic fuzzy environments using different subsections of the section 3 and a complex system with its configuration is shown in this section. The

constraint handling approach and the solution methodology are kept in section 4. The computational procedure of the proposed GA is described in subsection 4.1 with the help of a flow chart. Results and discussions of the numerical example are kept in subsection 5.1 and the sensitivity is analyzed in subsection 5.2 of the section 5. At last conclusion of the entire work with some future scopes is drawn in section 6.

## 2. Preliminaries

**2.1 Fuzzy Number:** A convex and normal fuzzy set [31] is defined as fuzzy number i.e., a fuzzy number is a special kind of fuzzy set.

**2.2 Intuitionistic fuzzy number:** An intuitionistic fuzzy number [33]  $\tilde{B}$  is an intuitionistic fuzzy set such that

- a) it is contained in the real line;
- b) it is normal i.e., there is at least one  $y_0 \in \mathbb{R}$  for which  $\mu_{\tilde{B}}(y_0) = 1$  (and  $\nu_{\tilde{B}}(y_0) = 0$ );
- c)  $\mu_{\tilde{B}}(\lambda y_1 + (1 - \lambda)y_2) \geq \min(\mu_{\tilde{B}}(y_1), \mu_{\tilde{B}}(y_2)) \forall y_1, y_2 \in \mathbb{R}, \lambda \in [0,1]$  that means it is convex for the membership function  $\mu_{\tilde{B}}(y)$ ;
- d)  $\nu_{\tilde{B}}(y) \leq \max(\nu_{\tilde{B}}(y_1), \nu_{\tilde{B}}(y_2)) \forall y_1, y_2 \in \mathbb{R}, \lambda \in [0,1]$  that means it is concave for non-membership function.

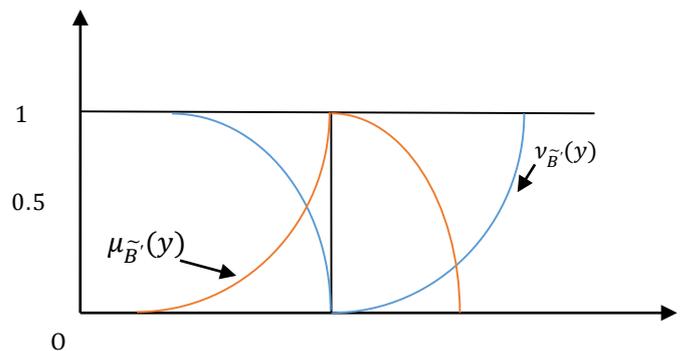


FIG. 1: INTUITIONISTIC FUZZY NUMBER ( $\tilde{B}$ )

## 2.3 Triangular Intuitionistic Fuzzy Number (TIFN)

An intuitionistic fuzzy set is said to be triangular intuitionistic fuzzy number (TIFN)  $\tilde{B}^i$  if the membership function ( $\mu_{\tilde{B}^i}(y)$ ) and non-membership function ( $\nu_{\tilde{B}^i}(y)$ ) are as follows:

$$\mu_{\tilde{B}^i}(y) = \begin{cases} \frac{y - e_1}{e_2 - e_1}, & \text{for } e_1 \leq y \leq e_2 \\ \frac{e_3 - y}{e_3 - e_2}, & \text{for } e_2 \leq y \leq e_3 \\ 0, & \text{otherwise} \end{cases}$$

$$v_{\tilde{B}^i}(y) = \begin{cases} \frac{e_2 - y}{e_2 - e'_1}, & \text{for } e'_1 \leq y \leq e_2 \\ \frac{y - e_2}{e'_3 - e_2}, & \text{for } e_2 \leq y \leq e'_3 \\ 1, & \text{otherwise} \end{cases}$$

where,  $e'_1 \leq e_1 \leq e_2 \leq e_3 \leq e'_3$  and for  $\mu_{\tilde{B}^i}(y) = v_{\tilde{B}^i}(y)$ ,  $\mu_{\tilde{B}^i}(y), v_{\tilde{B}^i}(y) \leq \frac{1}{2} \forall y \in R$ .

Triangular intuitionistic fuzzy number [34] is written as  $(e_1, e_2, e_3; e'_1, e_2, e'_3)$ .

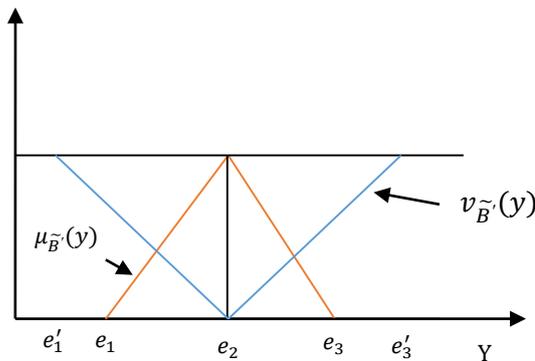


FIG. 2: TRIANGULAR INTUITIONISTIC FUZZY NUMBER (TIFN)

**2.3.1 Transformation rule:** TIFN,  $\tilde{B}^i = (e_1, e_2, e_3; e'_1, e_2, e'_3)$  reduces to

- (i) a triangular fuzzy number (TFN),  $\tilde{A} = (e_1, e_2, e_3)$  if  $e_1 = e'_1, e_3 = e'_3$  and  $v_{\tilde{B}^i}(z) = 1 - \mu_{\tilde{B}^i}(z)$
- (ii) real interval  $[e_1, e_3]$  if  $e'_1 = e_1$  and  $e_3 = e'_3$ .
- (iii) a real number 'e' if  $e'_1 = e_1 = e_2 = e_3 = e'_3 = e$ .

**2.4 Graded mean integration method for triangular fuzzy number (TFN)**

According to [30] graded mean integration formula for the triangular fuzzy number (TFN)  $\tilde{B} = (e_1, e_2, e_3)$  is given by  $G(\tilde{B}) = \frac{e_1 + 2e_2 + e_3}{4}$ .

**2.4.1 Graded mean integration method for TIFN:** Let  $\tilde{B}^i = (e_1, e_2, e_3; e'_1, e_2, e'_3)$  be triangular intuitionistic fuzzy number. Then graded mean integration formula for membership and non-membership functions are given by

$$G_\mu(\tilde{B}^i) = \frac{e_1 + 2e_2 + e_3}{4}, \quad G_\nu(\tilde{B}^i) = \frac{e'_1 + 2e_2 + e'_3}{4}$$

Now, taking the mean of  $G_\mu(\tilde{B}^i)$  and  $G_\nu(\tilde{B}^i)$ , the graded mean integration formula of  $\tilde{B}^i$  becomes as follows:

$$G_{av}(\tilde{B}^i) = \frac{e_1 + 2e_2 + e_3 + e'_1 + 2e_2 + e'_3}{8}$$

**3. Model Formulation**

**3.1 Assumptions and notation**

Throughout the paper, the following assumptions and notation are employed.

**3.1.1 Assumptions**

- ❖ Component reliabilities are taken as intuitionistic fuzzy number.
- ❖ The probability of failure of any component does not depend on the failure of other components.
- ❖ All the control parameters and cost coefficients are taken as intuitionistic fuzzy number.

**3.1.2 Notation Symbols Descriptions**

m	subsystems' number
$z = (z_1, z_2, \dots, z_m)$	redundancy vector
$t_i, \tilde{t}_i$	reliability of $i^{\text{th}}$ crisp, intuitionistic fuzzy component
$t = (t_1, t_2, \dots, t_m)$	system's reliability vector
$Z_S(z), \tilde{Z}_S(z)$	system reliability in crisp and intuitionistic fuzzy environment
$g_j(z), \tilde{g}_j(z)$	$j^{\text{th}}$ constraint functions ( $j= 1, 2, \dots, n$ ) in crisp and intuitionistic fuzzy environment
$p_i, \tilde{p}_i$	volume of $i^{\text{th}}$ component in crisp and intuitionistic fuzzy cases
$q_i, \tilde{q}_i$	cost of $i^{\text{th}}$ component in crisp and intuitionistic fuzzy cases
$r_i, \tilde{r}_i$	weight of $i^{\text{th}}$ component in crisp and intuitionistic fuzzy cases
$P, \tilde{P}$	upper limit of volume constraint in crisp and intuitionistic fuzzy environments
$Q, \tilde{Q}$	upper limit of cost constraint in crisp and intuitionistic fuzzy environments

$R, \bar{R}$  upper limit of weight constraint in crisp and intuitionistic fuzzy environments

$b_j, \tilde{b}_j$  availability of  $j^{\text{th}}$  resource in crisp and intuitionistic fuzzy cases

$\theta$  region of feasibility

is being shown in Fig.3. The overall system reliability  $Z_S(z)$  is given below:

$$\begin{aligned} \text{Maximize } Z_S(z) = & (1 - Z_5(z))[1 - (1 \\ & - Z_1(z_1)Z_2(z_2))(1 - Z_3(z_3)Z_4(z_4)) \\ & + Z_5(z_5)(1 - (1 - Z_1(z_1))(1 \\ & - Z_3(z_3)))(1 \\ & - (1 - Z_2(z_2))(1 - Z_4(z_4)))] \end{aligned}$$

where,  $z = (z_1, z_2, \dots, z_5)$ .

### 3.2 Crisp Model

Maximize  $Z_S(z)$  (I)

subject to  $g_j(z) - b_j \leq 0, j = 1, 2, \dots, n$

where,  $z = (z_1, z_2, \dots, z_m), 1 \leq L_i \leq z_i \leq U_i, z_i$  is integer,  $i=1, 2, \dots, m, b_j$  is the  $j$ -th variable resource,  $j=1, 2, \dots, n$ .

$Z_S(z), g_j(z)$  and  $b_j$  are the respective reliability of the system, constraint at  $j$ -th position and resource at  $j$ -th position.

### 3.3 Fuzzy Model

Maximize  $\hat{Z}_S(z)$  (II)

subject to  $\hat{g}_j(z) - \hat{b}_j \leq 0, j = 1, 2, \dots, n$

where,  $z = (z_1, z_2, \dots, z_m), 1 \leq L_i \leq z_i \leq U_i, z_i$  is integer  $i=1, 2, \dots, m; b_j$  is the  $j$ -th variable resource,  $j=1, 2, \dots, n$ .

$\hat{Z}_S(z), \hat{g}_j(z)$  and  $\hat{b}_j$  are the respective fuzzy reliability of the system, fuzzy constraint at  $j$ -th position and fuzzy resource at  $j$ -th position.

### 3.4 Intuitionistic Fuzzy Model

Maximize  $\tilde{Z}_S(z)$  (III)

subject to  $\tilde{g}_j(z) - \tilde{b}_j \leq 0, j = 1, 2, \dots, n$

where,  $z = (z_1, z_2, \dots, z_m), 1 \leq L_i \leq z_i \leq U_i, z_i$  is integer  $i=1, 2, \dots, m; b_j$  is the  $j$ -th variable resource,  $j=1, 2, \dots, n$ .

$\tilde{Z}_S(z), \tilde{g}_j(z)$  and  $\tilde{b}_j$  are the respective fuzzy reliability of the system, fuzzy constraint at  $j$ -th position and fuzzy resource at  $j$ -th position.

### 3.5 Complicated/Complex System

A system, consisting of five subsystems ( $m=5$ ) with three nonlinear and non-separable constraints ( $n=3$ ) is being considered. This complicated system

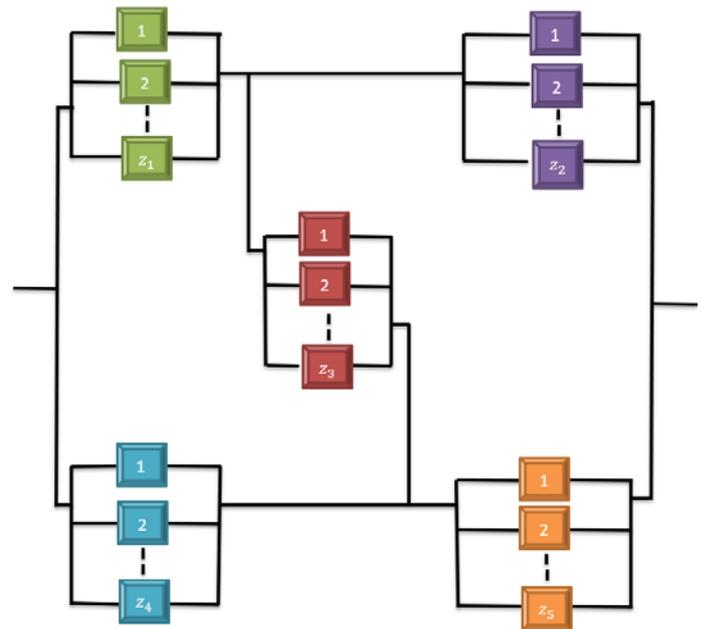


Fig. 3: Complex bridge system

### 4. Solution Procedure

In this paper, we looked at a constrained optimization problem. So, in order to tackle this problem, we must first deal with the limits. There are various constraint handling approaches. The Big-M penalty function technique [8] is proven to be very successful among them. For each of the infeasible solutions, a huge positive integer M is assigned to the goal value for minimizing the problem. For a maximization problem -M is set instead of M. Equation (IV) shows the implementation of Big-M Penalty technique for maximization problem.

$$\text{Maximize } G_{av}(\tilde{Z}_S(z)) = \begin{cases} G_{av}(\tilde{Z}_S(z)) & \text{if } z \in \theta \\ -M & \text{if } z \notin \theta \end{cases} \quad (IV)$$

where,  $\theta = \{z: G_{av}(\tilde{g}_j(z_1, z_2, \dots, z_m)) \leq G_{av}(\tilde{b}_j), j = 1, 2, \dots, n\}$  represents the region of feasibility;  $G_{av}(\mathbf{I})$  is the Graded mean integration value of the intuitionistic fuzzy number  $\mathbf{I}$ .

The problem (IV) is an optimization problem with discrete variables that is very nonlinear. As a result,

solving this problem analytically has grown difficult. Furthermore, the gradient-based method or the indirect search approach cannot be used since these methods require the decision variables to be continuous [16]. Therefore, heuristic/meta-heuristic algorithm becomes the necessary tool to obtain the solution of the problem.

To tackle the optimization problem in this paper, actual coded elitist GA is employed.

### 4.1 Genetic Algorithm

GA [7,19] is a stochastic search and optimization strategy based on natural genetics and the evolutionary principle of “Survival of the Fittest” that follows two simple principles:

- a) “If an above-average offspring is formed by genetic processing, it will survive longer than an average individual and hence have more opportunity to have kids with some of its qualities than an ordinary individual.”
- b) “However, if a below-average child is produced, it does not survive and is thus eliminated from the population.”

Some of GA's more well-known features are as follows:

- (i) GA searches the coding of a solution set rather than the solution itself.
- (ii) Rather than using derivatives or another auxiliary knowledge, GA uses payoff information.
- (iii) GA relies on reward information rather than derivatives or other auxiliary data.
- (iv) GA uses stochastic transformation rules rather than deterministic transformation procedures.

Figure 4 shows a block diagram of real coded elitist GA.

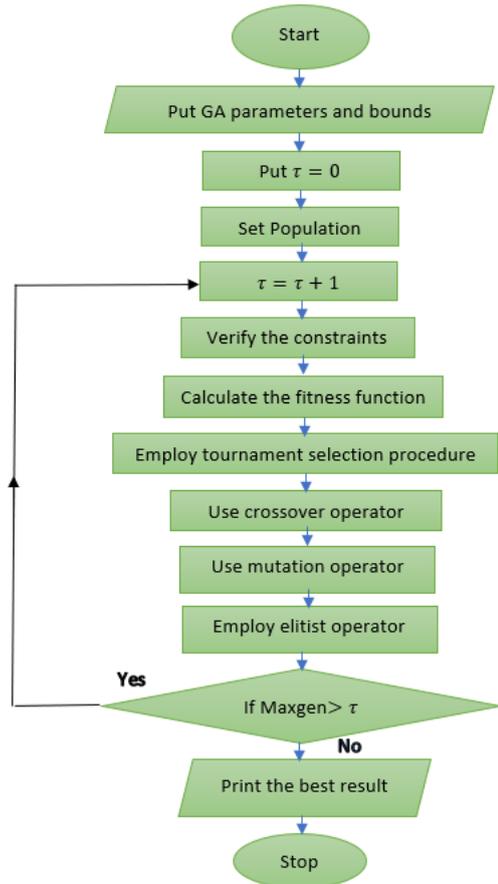


Fig. 4: Block diagram of proposed GA

The value of the objective function corresponds to the chromosome's fitness value. The approach uses tournament selection with two team, intermediate crossover, and mutation in one neighborhood because the variables are discrete. When the algorithm reaches a predetermined number of generations, it will be terminated.

### 5. Numerical Illustrations

**Example:** The redundancy allocation problem with respect to crisp atmosphere is as follows:

$$\text{Maximize } Z_5(z) = Z_5(z_5)(1 - (1 - Z_1(z_1))(1 - Z_3(z_3)))(1 - (1 - Z_2(z_2))(1 - Z_4(z_4))) + (1 - Z_5(z))[1 - (1 - Z_1(z_1)Z_2(z_2))(1 - Z_3(z_3)Z_4(z_4))]$$

subject to

$$p_1 \exp\left(\frac{z_1}{2}\right) z_2 + p_2 z_3 + p_3 z_4^2 + p_5 z_5 \leq P$$

$$q_1 \exp\left(\frac{z_1}{2}\right) + q_2 \exp(z_2) + q_3 z_3^2 + q_4 \left[ z_4^2 + \exp\left(\frac{z_4}{4}\right) \right] + q_5 \exp\left(\frac{z_5}{4}\right) \leq Q$$

$$r_2(z_2^2 + \exp(z_2)) + r_3z_3 \exp\left(\frac{z_3}{4}\right) + r_4z_1z_4^2 + r_5z_5^3 \leq R$$

$$(1,1,1,1,1) \leq (z_1, z_2, \dots, z_5) \leq (10,10,10,10,10)$$

where,  $Z_1(z_1) = t_1$ ,

$$Z_2(z_2) = 1 - (1 - t_2)^{z_2^2},$$

$$Z_3(z_3) = \sum_2^{z_3+1} \binom{z_3+1}{k} (t_3)^k (t_4)^{z_3+1-k},$$

$$Z_4(z_4) = 1 - (1 - t_5)^{z_4^2},$$

$$Z_5(z_5) = 1 - (1 - t_6)^{z_5^3}.$$

The crisp, fuzzy and intuitionistic fuzzy representations of the control parameters are kept in Table 1, Table 2 and Table 3 respectively.

**Table 1: Data for crisp model (L. Sahoo 2017)**

Control parameters	Values in Crisp environment
$p_1$	10
$p_2$	20
$p_3$	3
$p_5$	8
$q_1$	10
$q_2$	4
$q_3$	2
$q_4$	6
$q_5$	7
$r_2$	12
$r_3$	5
$r_4$	3
$r_5$	2
P	200
Q	310
R	520
$t_1$	0.975
$t_2$	0.75
$t_3$	0.88
$t_4$	0.12
$t_5$	0.70
$t_6$	0.85

**Table 2: TFN data and crispified value for problem (II)**

Control parameters	TFN representation	GMIV of TFN
$\hat{p}_1$	(7,10,11)	9.5
$\hat{p}_2$	(18,20,21)	19.75
$\hat{p}_3$	(1,3,4)	2.75
$\hat{p}_5$	(7,8,10)	8.25
$\hat{q}_1$	(8,10,11)	9.75
$\hat{q}_2$	(2,4,5)	3.75
$\hat{q}_3$	(1,2,4)	2.75
$\hat{q}_4$	(5.5,6,7)	6.125
$\hat{q}_5$	(6.5,7,8)	7.125
$\hat{r}_2$	(9,12,13)	11.5
$\hat{r}_3$	(3,5,5.5)	4.625
$\hat{r}_4$	(1.5,3,4.0)	2.875
$\hat{r}_5$	(1.5,2,3)	2.125
$\hat{P}$	(190,200,205)	198.75
$\hat{Q}$	(305,310,312)	309.25
$\hat{R}$	(515,520,523)	519.5
$\hat{t}_1$	(0.90,0.95,0.97)	0.9425
$\hat{t}_2$	(0.70,0.75,0.78)	0.745
$\hat{t}_3$	(0.84,0.88,0.90)	0.875
$\hat{t}_4$	(0.10,0.12,0.15)	0.1225
$\hat{t}_5$	(0.65,0.70,0.74)	0.6975
$\hat{t}_6$	(0.81,0.85,0.87)	0.845

**Table 3: TIFN data and its crispified value for problem (III)**

Control parameters	TIFN representation	GMIV of TIFN
$\tilde{p}_1$	(7,10,12;6,10,13)	7.75
$\tilde{p}_2$	(18,20,22;16,20,24)	18
$\tilde{p}_3$	(2,3,4;1.5,3,5)	2.3125
$\tilde{p}_5$	(7,8,10;6,8,11.5)	7.3125
$\tilde{q}_1$	(9,10,11;7.5,10,12.5)	8.75
$\tilde{q}_2$	(3,4,5;2.5,4,6)	4.875
$\tilde{q}_3$	(1,2,3;0.5,2,3.5)	4.0625
$\tilde{q}_4$	(5.5,6,7;4,6,8)	5.0625
$\tilde{q}_5$	(6.5,7,8;5,7,9)	6.0625
$\tilde{r}_2$	(10,12,14;9,12,15)	10.5
$\tilde{r}_3$	(3,5,5.5;2.5,5,6.5)	3.4375
$\tilde{r}_4$	(1.5,3,4.0;1,3,4.5)	1.875
$\tilde{r}_5$	(1,2,3;0.7,2,3.5)	1.375
$\tilde{P}$	(190,200,205;188,200,208)	192.875
$\tilde{Q}$	(305,310,312;299,310,316)	303.5
$\tilde{R}$	(515,520,525;513,520,527)	516.5
$\tilde{t}_1$	(0.90,0.95,0.97;0.93,0.95,0.99)	0.93875
$\tilde{t}_2$	(0.70,0.75,0.78;0.67,0.75,0.79)	0.7025
$\tilde{t}_3$	(0.84,0.88,0.90;0.82,0.88,0.94)	0.8475
$\tilde{t}_4$	(0.10,0.12,0.14;0.09,0.12,0.16)	0.10625
$\tilde{t}_5$	(0.66,0.70,0.74;0.65,0.70,0.76)	0.67625
$\tilde{t}_6$	(0.83,0.85,0.87;0.81,0.85,0.89)	0.8275

**5.1 Results and Discussion**

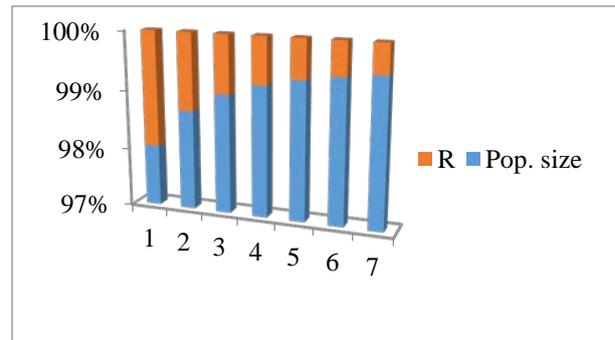
In a WINDOWS environment, the real programmed genetic algorithm is implemented in C. For each of the environments, 30 independent runs are considered in order to obtain maximum system reliability of the considered problem. Population size (200), maximum number of generations (200), Probability of crossover (0.85) and probability of mutation (0.15) are the GA parameters, employed in this study.

All of these criteria have been taken into account based on [18].

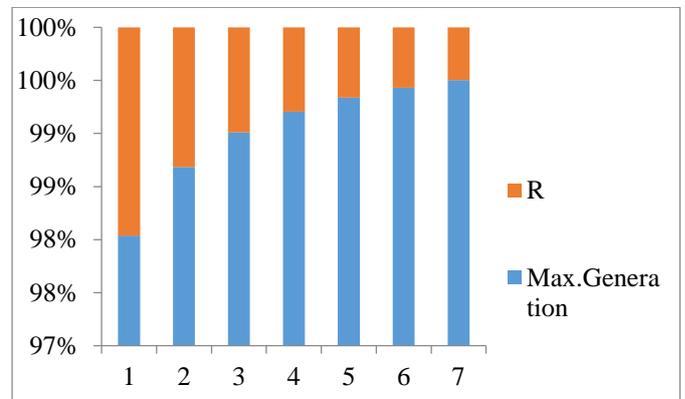
Table 4 shows a comparison of computational outcomes for crisp, fuzzy and intuitionistic fuzzy atmospheres.

Atmosphere	Vector of redundancy ( $z_1, z_2, z_3, z_4, z_5$ )	System reliability ( $Z_s$ )	Elapsed time of computation
Crisp	(1,3,4,3,2)	0.9999845	0.30 seconds
Fuzzy	(1,3,4,3,2)	0.99999634	0.0695 seconds
Intuitionistic fuzzy	(1,3,4,3,2)	0.99999821	0.07 seconds

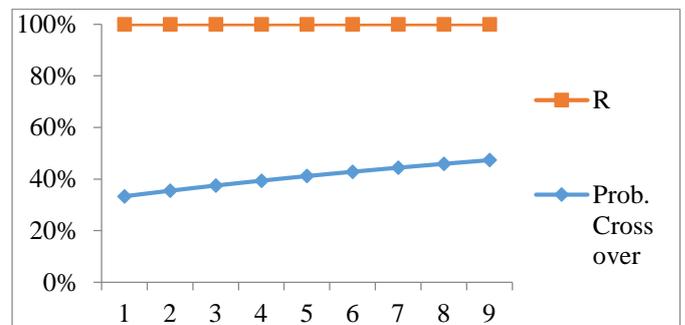
**Table 4: Comparison of results in crisp, fuzzy and intuitionistic fuzzy atmospheres**



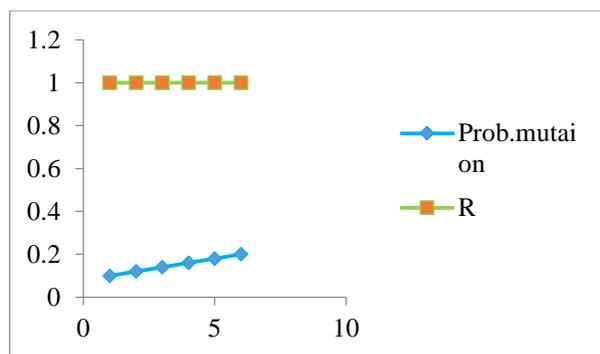
**Fig. 5: Behavior of system reliability w. r. t. population size for TIFN case**



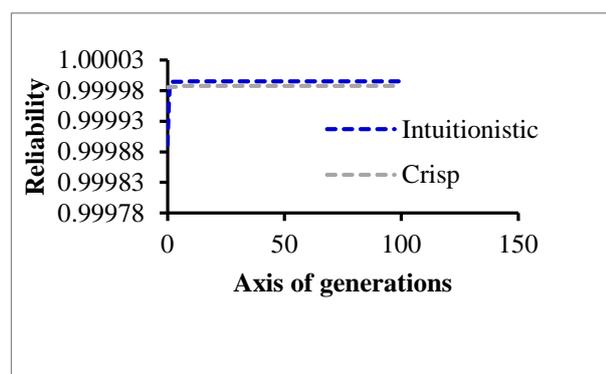
**Fig. 6: Behavior of system reliability w.r.t. maximum number of generations for TIFN case**



**Fig. 7: Behavior of system reliability w.r.t. probability of crossover for TIFN case**



**Fig. 8:** Behavior of system reliability w.r.t. probability of mutation for TIFN case



**Fig. 9:** History of convergency w.r.t. crisp and intuitionistic fuzzy data

### 5.2 Sensitivity Analysis

The maximum reliability of the system is obtained for the population size 200 in imprecise environment as shown in figure 5. Also, figure 6 shows the variations of the generations and the corresponding reliabilities of the system. In this paper, the maximum number of generations is taken as 200 for the increment of system reliability. According to figure 7, we see that the system reliability is uniform with respect to probability of crossover. The stability of the reliable system is depicted in figure 8. Figure 8 provides the guarantee of convergency of the optimality of the reliable system with respect to mutation operator. The history of convergency in precise and imprecise environments is shown in figure 9. From this figure we can say that the system reliability in imprecise environment overtakes the system reliability in crisp environment.

### 6. Conclusions

This study considers a complicated reliable system in crisp, fuzzy and intuitionistic fuzzy environments. The imprecision is represented with a fuzzy number that is intuitive. We get a nonlinear programming issue when the imprecise model is converted to a crispified model using an enhanced variant of the graded mean integration approach.

The problem at hand is solved using a real-coded elitist genetic algorithm and a penalty approach. From this study, we obtain the maximum of the objective function in intuitionistic fuzzy environment. We, also observe that consideration of fuzzy environment to the taken problem yields better objective value in comparison to crisp environment. The solution's sensitivities are graphed in terms of the maximum number of generations, population size, crossover probability, and mutation probability. The proposed algorithm can be used to find the global optimum for higher dimensional nonlinear integer programming problems. For further investigation, one can implement other imprecise environments and our suggested extended graded mean integration method for differently designed reliable systems. Other heuristic methods, such as PSO, DE, ABC and SA, may also be used to solve this problem as well as newly designed real-world problems.

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