Review Article

Journal of Scientific Enquiry
Vol: 2(2), Pages: 46-52

Why the photons don't have mass but the electrons do: An introduction to Higgs Mechanism

Sahazada Aziz*

Department of Physics, Ramananda Centenary College, Laulara-723151, Purulia, West Bengal, India

Received: 30.11.2022; accepted: 26.12.2022; published online: 31.12.2022

Abstract

In the present work, we are going to shed light on the very concept of mass of elementary particles. All the known elementary particles of nature were massless before electroweak symmetry breaking (EWSB). After EWSB some particles like electrons, quarks, etc., and short-ranging weak gauge bosons become massive whereas the particles like photons and gluons remain massless. The massive particles get their mass via Higgs Mechanism. In this article, we review the basics of local gauge invariance, quantum electrodynamics, the problem of local gauge invariance of massive gauge boson, Goldstone mechanism, and Higgs mechanism in Standard Model (SM) of electroweak theory and try to find out why some particles have mass and some don't. We also investigate why the masses of the elementary particles are so different from each other. An attempt has also been made to make the presentation of the paper as pedagogical as practicable. Though, mathematical justifications have been given at appropriate places where the theory demands more rigor. The concepts have been developed with the help of some toy models. Then they are applied to real-life problems. All the theories explained in this article have been established by accelerator and collider experiments, the final one being the discovery of the Higgs scalar itself in 2012. Furthermore, original references, as well as classic references, have been cited at appropriate places for the scope of further study by interested readers.

Keywords: Elementary particle physics, Quantum field theory, Standard Model, Higgs mechanism, Gauge symmetry

1. Introduction

There was a point of time where it was believed that the universe was significantly symmetric than that of at present. The four fundamental interact e.g., Gravitational, strong, electromagnetic, and weak interactions were expected to be unified, and the physics of that time might theoretically be explained by a single unified theory, albeit that has not been fully established. The strength of the interactions, characterized by coupling constants, depends on the energy scale (or distance scale) of the study. As time passes after Big Bang, the Universe cools down and the strength of the forces becomes different from one another. At the GUT (Grand Unified Theory) scale, when the energy was around 10^{16} GeV, strong, electromagnetic, and weak interactions were of the same strength. At $10^{-12} s$ after the Big Bang, when the energy of the Universe was around 100 GeV, another symmetry breaking took place when the electromagnetic and weak interactions became decoupled from each other. This symmetry breaking is called Electroweak symmetry breaking (EWSB). Before EWSB all the known particles of our nature were massless. After this symmetry breaking the matter building particles (quarks and leptons) and some of the messenger particles (Weak force carriers W^+, W^-, Z^0) became massive, whereas some messenger particles i.e., photons (γ) and gluons (g) remained massless. Apart from gravitational interactions, which is explained by Einstein's classical theory- General Theory of Relativity (GTR) the physics of the remaining three interactions have their standard quantum field theoretical models which are Quantum Chromodynamics (QCD) that explains strong interactions and Standard Model (SM) of Electroweak theory that explains the unified theory of weak and electromagnetic interactions. Interested readers can find ample exposition of the subject in references [1-7]. The Electroweak theory was developed by Sheldon Glashow, Steven Weinberg, and Abdus Salam during 1961-67 [8-10]. According to these theories, all matter building particles are spin ½ fermions (6 quark flavors plus their antiparticles in 3 color variants, 6 leptons plus their anti-leptons totaling 48 fermions) and the interactions are mediated among them by spin-1 gauge bosons (W^+, W^-, Z^0 for the weak force, 8 gluons for strong force and photon for electromagnetic force). But the main hurdle to formulate a consistent quantum gauge theory of massive particles is that it is non-renormalizable (a quantum field theory plagued by infinite terms). An older concept

*Electronic mail: sahazada@outlook.com

 DOI: https://doi.org/10.54280/jse.222102

developed, following Goldston's theorem, by Englert, Brout [11], Higgs [12], Guralnik, Hagen, and Kibble [13]: historically known as the Higgs mechanism came to the rescue. The theory also proposes a new spin-0 scalar particle known as Higgs boson (also famously known as the God particle in popular culture!), which is responsible for the mass-ness of other fermions and massive gauge bosons. So, we have got 61 elementary particles in total which constitutes the galaxies, stars, planets, living organisms, all chemical elements in periodic table, and so on. These are all normal matter in that the other 95% of the universe is made of Dark matter and Dark energy on which we don't wish to delve into in this article. This article is an account of what the Higgs mechanism really is and how mass is generated. It is noteworthy that all the predictions of SM have been verified and established with astounding accuracy. W^+, W^-, Z^0 gauge bosons were discovered in 1983 in CERN by groups led by Carlo Rubbia and Simon van der Meer. The last bit of the puzzle- the Higgs particle was finally discovered in CERN Large Hadron Collider in 2012.

This article is organized as follows. In Sec.2 we have discussed local gauge invariance and the quantum theory of light-matter interactions i.e., Quantum Electrodynamics (QED). In Sec.3 we schematically present the main features of electroweak theory and try to identify the problems of realization of local gauge invariance. Some examples of spontaneous symmetry breaking are given in Sec.4. In Sec.5 we have sketched the fully-fledged theory of SSB in SM and also explain the mass generation of electrons. Finally in Sec.6 we have some conclusions and discussions

2. Local Gauge invariance

Gauge invariance is a concept which dates back to the early twentieth century. But it gains immense popularity amongst particle physicists when 't Hooft showed [14] that Quantum field theories that are locally gauge invariant are renormalizable which means that the bad infinite parts can be tamed, rendering the theory physically acceptable. Here we are going to show the concept in a simple manner. From our knowledge of electromagnetism, we know that the value of an electric field $\vec{E}$ and magnetic field $\vec{B}$ remains unchanged under the simultaneous transformations of the potentials ($\phi, \vec{A}$):

$$\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\lambda, \phi \to \phi' = \phi - \frac{\partial \lambda}{\partial t} \quad (1)$$

Here, λ is a scalar. Eq.[1] can be compactly written as

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu \lambda \quad (2)$$

,where $\mu = 0, 1, 2, 3$ are the usual Lorentz indices. When λ depends on space-time i.e. $\lambda = \lambda(x)$, then the transformation is called local gauge transformation. Here

2.1 Local gauge invariance and QED

Another great example of local gauge invariance can be cited from Quantum electrodynamics (QED), which is a quantum theory of light-matter (electron or positron) interaction [15]. The Dirac Lagrangian can be written as

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi, \quad (3)$$

where ψ is the electron field and m is the mass of an electron. The readers should observe from the Lagrangian that the concept of photon or light doesn't enter in the picture and the Lagrangian involves electron field only. Now, we demand that the Lagrangian is invariant under a local gauge (phase) transformation of the form

$$\psi(x) \to \psi(x)' = e^{i\alpha(x)}\psi(x). \quad (4)$$

We observe that it is not invariant due to the first term. To make it locally gauge invariant we promote the partial derivative to a “covariant” one i.e., the derivative term changes the same way as the field does:

$$D_\mu \psi \to D'_\mu \psi' = e^{i\alpha(x)}(D_\mu \psi). \quad (5)$$

This transformation holds if we define the covariant derivative as

$$D_\mu \equiv \partial_\mu + ieA_\mu \quad (6)$$

where the gauge field A_μ introduced here transform under the local gauge transformation as

$$A_\mu \to A'_\mu = A_\mu - 1/e\partial_\mu \alpha(x). \quad (7)$$

The transformation formulas (7) and (2) are identical, from which we conclude that the gauge field introduced in this theory is nothing but our old electromagnetic field! We started off our theory with electrons only, but the necessity of fulfillment of local gauge invariance condition forces us to include interaction with photon field! This concept is the cornerstone of the gauge theory of particle physics.

If we include kinetic term of the gauge field the QED lagrangian becomes

$$\mathcal{L}_{QED} = i\psi\gamma^\mu D_\mu \psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (8)$$

where the classical electromagnetic field strength tensor is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (9)$$

3. Problem of Local Gauge invariance in Glashow-Salam-Weinberg (GSW) Model

The type of theories where no interaction between gauge fields is there and the members of the gauge group commute with one another are called Abelian gauge theories. For example, QED is an Abelian gauge theory. On the other hand, GSW theory, which describes Electroweak physics, is a non-Abelian gauge theory. One startling difference between QED and GSW theory is that, in addition to the photon, the latter predicts massive gauge bosons which are responsible for the observed short-range of weak interaction. Before going to more thorough discussion of local gauge invariance of the theory let us give here a lightning review of the GSW model. GSW model is a $SU(2)_L \times U(1)_Y$ product group model where the indices L, Y denote left-handedness (Chirality) and Hypercharge respectively. Weak interaction violates the parity symmetry maximally. This interaction affects only the left-handed particle. To develop the theory left-handed leptons are taken in a weak isospin doublet

$$\chi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} \nu \\ e \end{pmatrix}_L. \quad (10)$$

The element in the upper and lower half has weak isospin quantum numbers $T^3 = \pm\frac{1}{2}$ and the doublet has $T = \frac{1}{2}$. The generators of isospin are taken as 2×2 Pauli matrices which satisfy the SU(2) Lie algebra:

$$\left[\frac{1}{2}\tau^i, \tau^j\right] = i\epsilon_{ijk}\frac{1}{2}\tau^k. \quad (11)$$

The weak current is given by

$$J^i_\mu = \bar{\chi}_L \gamma_\mu \frac{1}{2}\tau^i \chi_L, \quad (i = 1, 2, 3). \quad (12)$$

As the theory also describes electromagnetic interaction, we include the corresponding current:

$$J^{em}_\mu = Q\left(\bar{e}_L \gamma_\mu e_L + \bar{e}_R \gamma_\mu e_R\right). \quad (13)$$

As far as the net charge of the interacting particles is concerned J^3_μ, J^{em}_μ are neutral and J^1_μ, J^2_μ are charged currents.

The relation between electromagnetic and weak isospin current is given by

$$J^{em}_\mu = J^3_\mu + \frac{1}{2}J^Y_\mu. \quad (14)$$

Eq.14 translates a Gell-Mann Nishijima type relationship among the electric charge (Q) third component of isospin (T^3) and weak hypercharge (Y):

$$Q = T^3 + \frac{1}{2}. \quad (15)$$

In the preceding discussion, we have talked about the first generation of leptons only that can be extended for the other two generations in the same manner. The same discussion can also be carried out to quark sector also with little modifications.

We are now going to write down the terms in the Lagrangian of the leptonic sector of the standard model. QED has the interaction term of the form $-eJ^\mu A_\mu$ and the electromagnetic current has the form $Q\bar{\psi}\gamma^\mu\psi$. Note that It has three terms in a product form: coupling constant i.e., electric charge, current, and gauge field. Similarly, the interaction of the weak current with gauge fields can be written as $-gJ^{i\mu}W^i_\mu - \frac{g'}{2}J^{Y\mu}B_\mu$, where g, g' are coupling constants with the three gauge fields corresponding to gauge group $SU(2)_L$: W^i_μ (i=1,2,3) and one corresponding to gauge group $U(1)_Y$: B_μ.

In reality, we do not observe these massless gauge bosons but we observe four physical gauge fields: W^+, W^-, Z^0, A_μ. These two sets of fields are related with each other in a particular relationship.

We now take a pause here in order to recapitulate what have we have learned so far.

1. Local gauge invariance is a cornerstone in the development of a theory describing fundamental interaction.
2. QED is the quantum theory that describes the light-matter interaction. The demand of local gauge invariance automatically includes a massless gauge field which is nothing but a photon.
3. In QED we begin our discussion with massive electrons and the mass term is invariant under local gauge transformation. On the other hand, the issue of invariance of gauge boson mass term is not of any importance as the photon is massless and the range of the interaction is infinite.
4. GSW theory describes electroweak interaction. It predicts three massive plus one massless (photon) gauge boson.

3.1 Identifying the crux of the mass problem

The mass term of the gauge bosons (for example $\frac{1}{2}M^2B_\mu B^\mu$) is not invariant under a local gauge symmetry. So, we cannot put the mass term by hand. The same problem also persists in the fermion sector. Unlike QED, $SU(2)_L \times U(1)_Y$ is a chiral theory. So we cannot put a Dirac mass term, of the form $m\bar{\psi}\psi = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$, ab initio as it is not locally gauge symmetric; the left- and right-handed fields transform differently under such transformation.

The solution of the problem lies in the introduction of the concept of spontaneous symmetric breaking (SSB) and the Higgs mechanism which are the topic of our discussion in the following section.

4. Spontaneous Symmetry Breaking and Higgs Mechanism

4.1 A toy Model

Before proceeding to a more formal aspect of the Higgs mechanism let us first consider a toy model to understand it. The typical Lagrangian (Kinetic energy minus potential energy) for a spin-0 real scalar particle is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 \quad . \qquad (16)$$

where the first term is the KE term and the second one is the mass term. Note the negative sign in the mass term.

We now set $m = 0$ but add a potential term of the form

$$V(\phi) = \frac{1}{4}\lambda\phi^4 - \frac{1}{2}\mu^2\phi^2 \qquad (17)$$

which is known as phi-four potential.

The Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) \quad \frac{1}{4}\lambda\phi^4 + \frac{1}{2}\mu^2\phi^2 \qquad (18)$$

Note that, though the last term has a phi-squared part, it is not a mass term because of the positive sign.

The potential is the famous Mexican hat or wine-bottle potential which is depicted in Fig.1.

Due to the presence of the ϕ^4 term in the potential, it is called phi-four potential. The potential is a special one in that the ground state, i.e., the minimum value of the potential of Eq.17, is not at $\phi = 0$ but at another place. To find the minimum we equate Eq.17 to zero and find, $\phi_0 = \pm\sqrt{\frac{\mu^2}{\lambda}} = \pm v$ where v is the vacuum (ground state of a quantum field) expectation (like average) value or VEV of the scalar field ϕ. The VEV of a field is given by $v = \langle 0|\phi|0\rangle$.

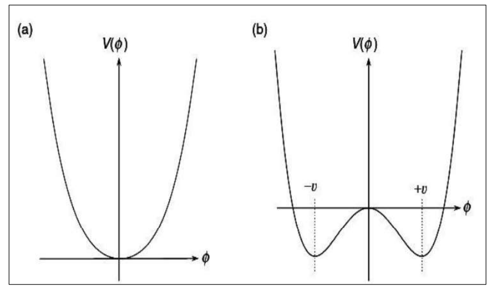

Fig.1 The one-dimensional potential as described above, For (a) μ^2 term positive and (b) μ^2 term negative. The second one is the Mexican-hat potential with degenerate minima at $\phi = \pm v$.

4.2 Spontaneous symmetry breaking

From the above brief discussion, we see that the configuration has two degenerate (i.e., repeated) minima at value ϕ_0. The potential looks very symmetric with respect to $\phi = 0$; It looks same when $\phi \rightarrow -\phi$: mirror or left-right symmetry. But will it remain symmetric with respect to the proper ground (vacuum) state given by $\phi_0 = \pm\sqrt{\frac{\mu^2}{\lambda}} = \pm v$? The answer is: NO. When we choose the ground state at any of the one VEV, the mirror symmetry breaks! So, the mirror symmetry is respected by the Lagrangian of the system but it is not maintained by the choice of ground state we make. We did nothing to break the symmetry. For example, no extra terms have been added by hand. Such a type of breaking of symmetry is termed spontaneous symmetry breaking (SSB).

4.3 SSB in ferromagnet

The concept of SSB has been borrowed from the field of condensed matter physics. For an illustration, we take a sample of Ferromagnet. Above a particular temperature, called Curie temperature (T_C), the spins are randomly oriented and the system possess a three-dimensional rotation symmetry. Such a system doesn't have Ferromagnetism; it is paramagnetic. When the temperature is decreased below T_C, all the spins become oriented to a particular direction inside a domain. The rotational symmetry is lost or said to be broken and the sample has now become

Ferromagnetic. We say that the symmetry is broken spontaneously.

4.4 Re-appearance of the mass term

We now shift the origin and express ϕ in terms of a new field ρ such that $\phi = \rho \pm \frac{\mu^2}{\lambda}$. We now have the same Lagrangian with a shift of origin:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)(\partial^\mu \rho) - \mu^2 \rho^2 \pm \mu\sqrt{\lambda}\rho^3 - \frac{1}{4}\lambda\rho^3. \quad (19)$$

The third and fourth terms represent interactions. But, note that the second term is a mass term for ρ field! We started our theory with fields with no mass, but the process of spontaneous symmetry breaking eventually gives mass to the shifted field. This concept is the first hint to Higgs mechanism.

5. Theory of SSB and Higgs Mechanism in the Standard Model

Before going to the actual theory it is worthy to state a famous theorem by Goldstone [17,18] which states that *When a continuous global symmetry is broken spontaneouly, massless fields appear. The numbers of such fields are equal to the broken generator of the symmetry group*. These bosons are called Nambu-Goldstone bosons (NGBs) or Goldstone bosons for brevity.

This theorem finds its place for the first time for the explanation of Meissner Effect (magnetic flux exclusion by a superconductor) in Superconductivity by Anderson [19]. Peter Higgs implemented similar method in particle physics for the generation of weak gauge boson masses. But Higgs mechanism is different from Goldstone mechanism [17,20] in that the former concerns about spontaneous breaking of local gauge symmetry where the would-be Goldstone bosons do not appear at all. In other words they are "gauged away".

Before implementing the theory of SSB and Higgs mechanism in SM we should bear in mind the following points:

1. the Goldstone bosons should disappear in the theory.
2. W^+, W^- and Z^0 have mass. The masses are found to be $M_W \simeq 80.3 GeV, M_Z \simeq 91.2 GeV$.
3. Photon remains massless.
4. $M_W/M_Z = \cos\theta_W$, θ_W being the Weinberg angle.
5. To proceed with local gauge invariance the scalar field should be complex.

We have observed earlier how the condition of local gauge invariance promotes the ordinary derivative to a covariant one. In $SU(2)_L \times U(1)_Y$ theory we take the covariant derivative as

$$D_\mu = \partial_\mu + \frac{i}{2} g\vec{\tau}\cdot\vec{W}_\mu + ig'e\frac{Y}{2}B_\mu, \quad (20)$$

where g, g' are the coupling constants. We take the Higgs scalar field as a $SU(2)_L$ doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (21)$$

where, in terms of real scalar fields ϕ's

$$\phi^+ = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \text{ and } \phi^0 = \frac{\phi_3 + i\phi_4}{\sqrt{2}}. \quad (22)$$

The scalar field Lagrangian is given by

$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger (D^\mu\Phi) - V(\Phi) \quad (23)$$

where,

$$V(\Phi) = \mu^2(\Phi^\dagger\Phi) - \lambda(\Phi^\dagger\Phi)^2. \quad (24)$$

From Eq.24 we see that the degenerate minima are given by

$$\Phi^\dagger\Phi|_{min} = \frac{\mu^2}{2\lambda} \text{ or}$$
$$\frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{\mu^2}{2\lambda} \quad (25)$$

We choose the VEVs in the three directions to zero and set the VEV of ϕ_3 as

$$\langle 0|\phi_3|0\rangle = v = \sqrt{\frac{\mu^2}{\lambda}}. \quad (26)$$

To do perturbative physics we expand the scalar field around VEV v as $\phi_3 = v + H$, where H is the physical neutral Higgs scalar field. In a chosen *unitary gauge,*

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H \end{pmatrix}. \quad (27)$$

If we put Eqs.21,27 in Eq.23 we finally obtain

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu H \partial^\mu H + \frac{1}{4}g^2(H^2 + 2vH + v^2)W_\mu^+ W^\mu$$

$$+\frac{1}{8}(g^2+g'^2)(H^2+2vH+v^2)Z_\mu Z^\mu+\mu^2 H^2+\frac{\lambda}{4}(H^4+4vH^3). \quad (28)$$

The masses of $W^{\pm}$ and Z can be read off from Eq.28 as

$$M_W=\frac{1}{2}gv \text{ and}$$

$$M_Z=\frac{1}{2}(g^2+g'^2)^{1/2}v=\frac{1}{2}\frac{gv}{cos\theta_W}. \quad (29)$$

We also observe the important result that we have been looking for: the mass of photon field A_μ is zero. The Higgs boson mass is given by

$$m_H=\sqrt{-2\mu^2}=\sqrt{-2\lambda v^2}. \quad (30)$$

In summary, we can interpret the result as follows. Due to the spontaneous symmetry breaking from $SU(2)_L\times U(1)_Y\rightarrow U(1)_{em}$ one massive scalar and three massless Nambu-Goldstone boson emerge; three of them are, in plain english, "eaten-up" by three gauge bosons W^+, W^-, Z^0 which makes them massive. In other words these three scalar degrees of freedom become longitudinal mode of the gauge bosons making them massive vector bosons.

5.1 Electron mass generation

All the leptons, including electron, and quarks in the SM get their mass by interacting with Higgs field. Such interaction is termed as Yukawa interaction. Yukawa interaction of electron can be written as

$$\mathcal{L}_Y(e)=-G_e[\chi_L\Phi e_R+e_R\Phi^\dagger\chi_L] \quad (31)$$

where, G_e is the Yukawa coupling constant of electron with Higgs field. Higgs field interact with each leptons and quarks with different coupling strengths and so the mass of the particles are different from each other.

Putting Eq.27 in Eq.31 we have

$$\mathcal{L}_Y(e)=\frac{G_e}{\sqrt{2}}(v+H)[\bar{e}_L e_R+\bar{e}_R e_L]. \quad (32)$$

From this we identify the mass term of electron as $m_e=\frac{G_e v}{\sqrt{2}}$.

6. Conclusion and Discussions

In this article, an attempt has been made to understand what is the real process responsible for mass generation. To realize the physics thoroughly we had to introduce and understand several physical concepts. These are local gauge invariance, quantum electrodynamics, electroweak theory, Goldstone and Higgs mechanism, and spontaneous symmetry breaking. We finally got our answers to the questions that have asked in the title of the article itself.

Furthermore, the physics of quarks, gluons, and their interaction- Quantum Chromodynamics (QCD) and quark sector of the SM Lagrangian has been skipped deliberately to emphasize the main issue in this compact and brief report. We have only discussed the leptonic sector; though the same formalism can be extended to the quark sector with little modifications. Another point is worth noting here that *neutrinos* are considered massless in electroweak SM. But they possess very tiny, about one-billionth of electron mass (as per cosmological constraints), mass- a definitive and successful theory of its mass generation is still under consideration.

In conclusion, we reemphasize the conceptual meaning of mass in Standard Model. Mass does not mean the matter content of a particle or field, all the elementary particles being point. It is something more like a measure of inertia. The whole universe is filled with Higgs field. All elementary particles interact with this field with different intensities i.e., their coupling constants are different; the stronger the binding, the more is the mass. These particles and Higgs particle got their mass only after electroweak symmetry breaking. Photon doesn't interact with Higgs scalar in the same manner as the massive weak bosons do, thus remaining massless after symmetry breaking.

References

[1] Matthew D. Schwartz, Quantum Field Theory and the Standard Model, Cambridge University Press (2014).

[2] Michael E. Peskin and Daniel V. Schroeder, An Introduction To Quantum Field Theory,, CRC Press (1995).

[3] Lewis H. Ryder, Quantum Field Theory, Cambridge University Press, 2nd edition (1996).

[4] Mark Thomson, Modern Particle Physics, Cambridge University Press (2010).

[5] Ian J.R. Aitchison and Anthony J.G. Hey, Gauge Theories in Particle Physics: A Practical Introduction, Vol-I & II, CRC Press, 4th edition (2021).

[6] David Griffiths, Introduction to Elementary Particles, Wiley-VCH, 2nd edition (2008).

[7] Francis Halzen and Alan D. Martin, Quarks & Leptons: An Introductory Course In Modern Particle Physics, Wiley (2008).

[8] S.L. Glashow, Partial-symmetries of weak interactions, Nuclear Physics. 22 (4) 579 (1961).

[9] S. Weinberg, A Model of Leptons, Physical Review Letters. 19 (21) 1264 (1967).

[10] A. Salam, N. Svartholm (ed.), Elementary Particle Physics: Relativistic Groups and Analyticity, Eighth Nobel Symposium. Stockholm: Almquvist and Wiksell. p. 367 (1968).

[11] F. Englert and R. Brout, Broken Symmetry and the Mass of Gauge Vector Mesons, Physical Review Letters. 13 (9) 321 (1964).

[12] P. W. Higgs, Broken Symmetries and the Masses of Gauge Bosons, Physical Review Letters. 13 (16) 508 (1964).

[13] G. S. Guralnik, C.R. Hagen and T.W.B. Kibble, Global Conservation Laws and Massless Particles, Physical Review Letters. 13 (20) 585 (1964).

[14] G. 't Hooft, Renormalization of massless Yang-Mills fields, Nuclear Physics B. 33 (1) 173 (1971). Renormalizable Lagrangians for massive Yang-Mills fields, 't Hooft, G. ,Nuclear Physics B. 35 (1) 167 (1971).

[15] Richard Feynman, QED: The Strange Theory of Light and Matter, Princeton University Press (1985).

[16] Y. Nambu, Quasiparticles and Gauge Invariance in the Theory of Superconductivity, Physical Review. 117 (3) 648 (1960).

[17] J. Goldstone, JField Theories with Superconductor Solutions, Nuovo Cimento. 19 (1) 154 (1961).

[18] J. Goldstone, Abdus Salam and Steven Weinberg, Broken Symmetries, Physical Review. 127 (3) 965 (1962).

[19] Y. Nambu, Quasiparticles and Gauge Invariance in the Theory of Superconductivity, Physical Review. 117 (3) 648 (1960).

[20] P. W. Anderson, Coherent Excited States in the Theory of Superconductivity: Gauge Invariance and the Meissner Effect, Physical Review110 (4) 827 (1958).