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Discount cash flow approach to the optimal producing policy of the economic production quantity model under trade credit

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Abstract
This paper extends an EOQ model to consider the optimum production quantity the EPQ model that is not only dependent on the inventory policy but also on firms' credit policy. Here, the conditions of using a discounted cash-flows (DCF) approach and trade credit depending on the quantity produced are discussed. We consider that if the production quantity is less than at which the delay in payments is permitted, the payment for the item must be made immediately. Otherwise, the fixed trade credit period is permitted. This paper incorporates all concepts of a discounted cash-flows (DCF) approach, trade credit and the quantity production inventory to generalize the EOQ model.

Keywords: Inventory; Trade credit; EPQ model; Discounted Cash-flows

1. Introduction

The Economic Production quantity (EPQ) model is a simple mathematical model to deal with inventory management issues in a production-inventory system. It is considered to be one of the most popular inventory control models used in the industry. Huang et al. [1] established an EPQ model under cash discount and delay in payment with different selling and purchasing costs to incorporate Teng [2], Chung and Huang [3], and Huang and Chung [4]. Huang [5] developed an EPQ model under trade credit contract and generalized Chung and Huangs [6] proposed model by considering higher selling price than purchasing cost. Hu and Liu [7] extended Chung and Huang [8] to the EPQ framework with shortages and unequal selling and purchasing prices.

In addition, Hou and Lin presented an EOQ inventory model for deteriorating items under trade credit. They obtained the optimal ordering and pricing policies and discussed the effects of inflation, deterioration, and permissible delay in payment [9]. Also, Chung [10] formulated an EPQ model under two-level trade credit, from the Huang [11] viewpoint, limited storage capacity and different selling and purchasing prices. Molamohamadi et al. [13] formulated an EPQ model of an exponentially deteriorating item with price-sensitive demand under trade credit, where shortages are considered. Moreover, they applied cuckoo search algorithm for solving the model and demonstrated the effectiveness of trade credit over the classical inventory system.

This paper is basically an extension of the work of Chung and Liao [14]. They developed an EOQ model under order-quantity-dependent trade credit and DCF approach to generalize Chung [15]. It is concerned the credit policy may seem as an alternative to price discounts because such policies are not thought to provoke competitors to reduce their prices and thus introduce lasting price reductions, or because such policies are traditional in the firms industry. We present the economic producing policies in the presence of trade credit using a discounted cash-flows (DCF) approach. We divide the study into three cases:

(a) Instantaneous cash-flows (the case of the EPQ model).

(b) Credit only on units in stock when $T_P \leq M$ (where T_P denotes the production time and M denotes the credit period).

(c) Credit only on units in stock when $T_P \geq M$.

On the other hand, the credit policy may seem as an alternative to price discounts because such policies are not thought to provoke competitors to reduce their prices and thus introduce lasting price reductions, or because such policies are traditional in the firms industry. Furthermore, Khouja and Mehrez (1996) investigate the effect of supplier credit policies on the optimal order quantity within the economic order quantity framework. The supplier credit policies addressed in Khouja and Mehrez [12] fall into two categories. One is that, supplier credit policies where terms are independent of the order quantity and the other is that, supplier credit policies where credit terms are linked to the order quantity. In the latter case, suppliers use favorable credit terms to encourage customers to order large quantities. In other words, the favorable credit terms apply only at large order quantities and are used in place of quantity discounts. In this regard, this paper is only concerned with the latter case.

The paper is organized as follows. Section 2 describes the mathematical model and section 3 presents the solution procedure of the optimal inventory cycle time and a useful algorithm. In section 4, a numerical example to illustrate the model and sensitivity analysis of the optimal solution with respect to parameters of the system is

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carried out. The paper ends with concluding remarks in section 5.

2. Mathematical model and analysis

In this section, the mathematical model of the inventory system in case of the EPQ model is presented. The fact, the present value of all future cash-flows in different cases is computed.

2.1. Notations

The following notations will be used throughout the paper:

T the inventory cycle time, which is a decision variable
T_P the production cycle time
c the purchase cost per unit
D the demand rate per unit time
P the production rate per unit time
h the out-of-pocket inventory-carrying costs as a proportion of the value of inventory per unit time
r the opportunity cost (i.e., the doscount rate) per unit time
A the average ordering cost per order (or set-up cost per production run) in dollars
M the credit period
W quantity at which the delay in payments is permitted

2.2. Assumptions

Next, the following assumptions are made to establish the mathematical inventory model.

1. The demand rate is known and constant with time.
2. The replenishment rate is known and uniform.
3. The ordering lead time is zero.
4. Shortages are not allowed.
5. Time horizon is infinite.
6. If $Q < W$, the delay in payment is not permitted. Otherwise, certain fixed trade credit period M is permitted. That is, $Q < W$ holds if and only if $T < W/D$.
7. During the credit period, the firm makes payment to the supplier immediately after use of the materials. On the last day of the credit period, the firm pays remain balance.

3. Model Formulation

Now, the discounted cash flows approach will be employed for the analysis of the optimal inventory policy in the presence of the trade credit in different cases.

The above assumption (6) lead to the two cases to discuss: (i) $M > \frac{W}{P}$ and $M \le \frac{W}{P}$.

Additionally, this paper is presented the economic producing policies in the presence of trade credit using a discounted cash-flows (DCF) approach. We divide the study into three cases:

(a) Instantaneous cash-flows (the case of the EPQ model).
(b) Credit only on units in stock when $T_P \le M$.
(c) Credit only on units in stock when $T_P \ge M$.

The present values of cash-flows in above three cases are discussed as follows:

Case 1: Instantaneous cash-flows (the case of the EPQ model)

Case 1 presents the DCF approach for the EPQ model under the assumption of instantaneous inventory holding costs. Therefore, the present value of the set-up cost can be shown as:

$$A + Ae^{-rT} + Ae^{-2rT} + \cdots \ldots \ldots \ldots \ldots = \sum_{n=0}^{\infty} Ae^{-nrT} = \frac{A}{1-e^{-rT}}. \qquad (1)$$

The present value of the out-of-pocket inventory-carrying cost can be illustrated as:

$$hc\left[\sum_{n=0}^{\infty} \int_0^{T_P} (P-D)te^{-r(t+nT)}dt + \int_{T_P}^{T} D(T-t)e^{-r(t+nT)}dt\right]$$
$$= \frac{hc(P-D)}{r^2(1-e^{-rT})} - \frac{hc(Pe^{-rDT/P} - De^{-rT})}{r^2(1-e^{-rT})} = \frac{hcP(1-e^{-rDT/P})}{r^2(1-e^{-rT})} - \frac{hcD}{r^2}. \qquad (2)$$

Where $T_P = \frac{DT}{P}$, and the present value of the purchase cost can be presented as:

$$cDT + cDTe^{-rT} + cDTe^{-2rT} + \cdots \ldots \ldots \ldots = \sum_{n=0}^{\infty} cDTe^{-nrT} = \frac{cDT}{1-e^{-rT}}. \qquad (3)$$

Furthermore, the present value of all future cash-flows in this case is:

$$PV_1(T) = \frac{hcP(1-e^{-rDT/P})}{r^2(1-e^{-rT})} - \frac{hcD}{r^2} + \frac{A+cDT}{1-e^{-rT}} \qquad (4)$$

Case 2: Credit only on units in stock when $T_P \le M$

Case 2 assumes the existence of the credit period M. During the credit period, the firm makes payment to the supplier immediately after the use of the materials. On the last day of the credit period, firm pays the remaining balance. Additionally, the credit period is greater than the replenishment cycle length in this case. The present value of the purchase cost can be computed as:

$$c\sum_{n=0}^{\infty} \int_0^{T_P} Pe^{-r(t+nT)}dt = \frac{cP(1-e^{-rDT/P})}{r(1-e^{-rT})}. \qquad (5)$$

Then the present value of all future cash-flows in this case is:

$$PV_2(T) = \frac{hcP(1-e^{-rDT/P})}{r^2(1-e^{-rT})} - \frac{hcD}{r^2} + \frac{rA + cP(1-e^{-rDT/P})}{r(1-e^{-rT})}. \qquad (6)$$

Case 3: Credit only on units in stock when $T_P \ge M$

Case 3 deals with a similar situation to case 2. During the credit period, the firm makes payment to the supplier immediately after the use of the materials. On the last day of the credit period, the firm pays the remaining balance. Additionally, the credit period is shorter than the replenishment length in this case. The present value of the purchase cost can be shown as:

$$cP\sum_{n=0}^{\infty}\left[\int_0^{M} e^{-r(t+nT)}dt + (T_P - M)e^{-r(M+nT)}\right] = \frac{cP(1-e^{-rM}) + rc(DT-PM)e^{-rM}}{r(1-e^{-rT})}. \qquad (7)$$

Furthermore, the present value of all future cash-flows in this case is:

$$PV_3(T) = \frac{hcP(1-e^{-rDT/P})}{r^2(1-e^{-rT})} - \frac{hcD}{r^2} + \frac{rA + cP(1-e^{-rM}) + rc(DT-PM)e^{-rM}}{r(1-e^{-rT})}. \qquad (8)$$

Now, we will study the two cases, respectively.

(i) **Suppose that** $M > W/P$

For this case, combining the cases discussed above, we have

$$PV_\infty(T)) = \begin{cases} PV_1(T); & \text{if } 0 < T < \frac{W}{D} & (i) \\ PV_2(T); & \text{if } \frac{W}{D} \le T < \frac{MP}{D} & (ii) \\ PV_3(T); & \text{if } \frac{MP}{D} \le T & (iii) \end{cases}$$

(9)

where $PV_\infty(T)$ is the present value of all future cash-flow cost. From the following lemma, we should find that $PV_1(T) > PV_2(T)$ if $T > 0$.

Lemma 1: $rTe^{rDT/P} - e^{rDT/P} + 1 > 0$ if $rT > 0$.

Proof. Let $h(x) = xe^{ax} - e^{ax} + 1 > 0$ for $x > 0, 0 < a < 1$. Then we have $h'(x) = axe^{ax} + (1-a)e^{ax} > 0$ for $x > 0$. So $h(x)$ is increasing on $x \ge 0$. We get $h(x) > h(0) = 0$ for $x > 0$. Let $x = rT$ and $a = D/P$. We get $rTe^{rDT} - e^{rDT} + 1 > 0$ if $rT > 0$. We have completed the proof.

From Eqs. (4) and (6), we have

$$PV_1(T) - PV_2(2) = \frac{cDe^{-rDT/P}}{r(1-e^{-rT})}\left(rTe^{rDT/P} - e^{rDT/P} + 1\right) \quad (10)$$

(ii) **Suppose that** $M \le W/P$

When $M \le W/P, PV_\infty(T)$ can be expressed as follows:

$$PV_\infty(T) = \begin{cases} PV_1(T); & \text{if } 0 < T < \frac{W}{D} & (i) \\ PV_3(T); & \text{if } \frac{W}{D} \le T & (ii) \end{cases}$$

(11)

From the following lemma, we get that $PV_1(T) > PV_3(T)$ if $T_P \ge M$.

Lemma 2: $rDT(1-e^{-rM}) - Pe^{-rM}(e^{rM} - rM - 1) > 0$ if $T_P \ge M$.

Proof: Let $k_1(T) = rDT(1-e^{-rM}) - Pe^{-rM}(e^{rM} - rM - 1)$ for $T > 0$, then we have $k'(T) = rD(1-e^{-rM}) > 0$. Hence $k_1(T)$ is increasing on $T > 0$. Hence, $k_1(MP/D) = 0$, we get that $k_1(T) \ge k_1(MP/D) = 0$ for $T_P \ge M$ (i.e., $T \ge MP/D$). Consequently, $rDT(1-e^{-rM}) - Pe^{-rM}(e^{rM} - rM - 1) > 0$ if $T_P \ge M(T \ge MP/D)$. This completes the proof.

From Eqs. (4) and (8), we have

$$PV_1(T) - PV_2(T) = \frac{rDT(1-e^{-rM}) - Pe^{-rM}(e^{rM} - rM - 1)}{r(1-e^{-rT})}$$

(12)

and Lemma 2 implies that $PV_1(T) > PV_3(T)$ if $T_P \ge M$. Since $PV_2(MP/D) = PV_3(MP/D)$ and $PV_1(W/D) > PV_2(W/D), PV_\infty(T)$ is continuous except $T = W/D$.

4. The optimization procedure

Our objective is to minimize the present value of all future cash-flow cost $PV_\infty(T)$. That is,

$$\begin{cases} \text{Minimize } PV_\infty(T) \\ \text{Subject to } T > 0 \end{cases}$$

For this, the first derivative of $PV_1(T)$ with respect to T is given as:

$$\frac{dPV_1(T)}{dT} = \frac{hcDe^{-rDT/P}}{r(1-e^{-rT})} - \frac{hcPe^{-rT}(1-e^{-rDT/P})}{r(1-e^{-rT})^2} + \frac{cD}{1-e^{-rT}} - \frac{(A+cDT)re^{-rT}}{(1-e^{-rT})^2} \quad (13)$$

We can obtain, by setting $\frac{dPV_1(T)}{dT} = 0$ and simplifying the resulting equation, the optimal cycle time T_1^* as the unique non-negative solution of the following equation:

$$hcDe^{-\frac{rDT}{P}}(1-e^{-rT}) - hcPe^{-rT}\left(1-e^{-rDT/P}\right) + cDr(1-e^{-rT}) - (A+cDT)r^2e^{-rT} = 0.$$

(14)

We define

$$f_1(T) = hcDe^{-\frac{rDT}{P}}(1-e^{-rT}) - hcPe^{-rT}\left(1-e^{-rDT/P}\right) + cDr(1-e^{-rT}) - (A+cDT)r^2e^{-rT}.$$

(15)

Both $f_1(T)$ and $PV_1(T)$ have the same sign and domain. It is easy to obtain the following derivative

$$\frac{df_1(T)}{dT} = \frac{r}{P}\left[-hcD^2e^{-\frac{rDT}{P}}(1-e^{-rT}) + hcPe^{-rT}\left(1-e^{-rDT/P}\right) + (A+cDT)P^2e^{-rT}\right]$$
$$\ge \frac{r}{P}\left[-hcP^2e^{-\frac{rDT}{P}}(1-e^{-rT}) + hcP^2e^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) + (A+cDT)\mathrm{Pr}^2\ e^{-rT}\right]$$
$$\ge \frac{r}{P}\left[hcP^2\left(e^{-rT} - e^{-\frac{rDT}{P}}\right) + (A+cDT)\,\mathrm{Pr}^2\,e^{-rT}\right] > 0. \quad (16)$$

Furthermore, we have $f_1(0) = -(A+cDT)r^2 < 0$. Based upon the arguments, we sure that the solution to (4) not only exists but also is unique, which also implies that,

$$\frac{dPV_1(T)}{dT} = \begin{cases} < 0 & \text{if} \quad T < T_1^* \\ = 0 & \text{if} \quad T = T_1^* \\ > 0 & \text{if} \quad T > T_1^* \end{cases} \quad (17)$$

In similar argument as above, the optimal cycle time T_2^* and T_3^*, obtained by setting the derivative of Eqs. (4) and (6) with respect to T equal to 0, respectively, is the root of the following equation:

$$(h+r)cDe^{-\frac{rDT}{P}}(1-e^{-rT}) - (h+r)cPe^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) - Ar^2e^{-rT} = 0 \quad (18)$$

And

$$hcDe^{-\frac{rDT}{P}}(1-e^{-rT}) - hcPe^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) + cDre^{-rM}(1-e^{-rT}) - cPre^{-rT}(1-e^{-rM}) - Ar^2e^{-rT} - c(DT-PM)r^2e^{-r(M+T)} = 0. \quad (19)$$

Eqs. (18) and (19) are the optimality condition of Eqs. (6) and (8), respectively. Let

$$f_2(T) = (h+r)cDe^{-\frac{rDT}{P}}(1-e^{-rT}) - (h+r)cPe^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) - Ar^2e^{-rT} \quad (20)$$

And

$$f_3(T = hcDe^{-\frac{rDT}{P}}(1-e^{-rT}) - hcPe^{-rT}\left(1-e^{-rDT/P}\right) + cDre^{-rM}(1-e^{-rT}) - cPre^{-rT}(1-e^{-rM}) - Ar^2e^{-rT} - c(DT-PM)r^2e^{-r(M+T)}. \quad (21)$$

Then

$\frac{df_2(T)}{dT} = \frac{r}{P}\Big[-(h+r)cD^2e^{-\frac{rDT}{P}}(1-e^{-rT}) + (h+r)cP^2e^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) + AP^2e^{-rT}\Big]$

$\geq \frac{r}{P}\Big[-(h+r)cP^2e^{-\frac{rDT}{P}}(1-e^{-rT}) + (h+r)cP^2e^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) + APr^2e^{-rT}\Big]$

$\geq \frac{r}{P}\Big[(h+r)cP^2\left(e^{-rT} - e^{-\frac{rDT}{P}}\right)\Big] > 0.$

(22)

Also,

$\frac{df_3(T)}{dT} = \frac{r}{P}\Big[-hcD^2e^{-\frac{rDT}{P}}(1-e^{-rT}) + hcP^2e^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) + A\,\mathrm{Pr}^2\,e^{-rT}$
$+cP^2re^{-rT}(1-e^{-rM}) + c\mathrm{Pr}^2\,(DT-PM)e^{-r(M+T)}\Big]$

$\geq \frac{r}{P}\Big[-hcP^2e^{-\frac{rDT}{P}}(1-e^{-rT}) + hcP^2e^{-rT}\left(1-e^{-\frac{rDT}{P}}\right) + A\mathrm{Pr}^2\;e^{-rT}$
$+cP^2re^{-rT}(1-e^{-rM}) + cP^2(DT-PM)e^{-r(M+T)}\Big]$

$\geq \frac{r}{P}\Bigg[hcP^2\left(\sum_{i=0}^{\infty}\left(1-\left(\frac{D}{P}\right)^i\right)\frac{(-rT)^i}{i!}\right) + A\mathrm{Pr}^2\;e^{-rT}$
$+cP^2re^{-rT}\left(1-(1+rM)\left(\sum_{i=0}^{\infty}\frac{(-rM)^i}{i!}\right)\right) +$
$c\mathrm{Pr}^2\;DTe^{-r(M+T)}\Bigg] > 0,$ (23)

where $1-(1+rM)\left(\sum_{i=0}^{\infty}\frac{(-rM)^i}{i!}\right) = 1-1-rM+rM+rM^2-\frac{rM^2}{2}-\cdots\ldots\ldots > 0.$ Consequently, $f_i(T)(i=2,3)$ is also increasing on $(0,\infty)$ and we have $f_i(0)<0(i=2,3)$, then

$$\frac{dPV_i(T)}{dT} = \begin{cases} <0 & \text{if } T<T_i^* \\ =0 & \text{if } T=T_i^* \text{ for } i=1,2,3. \\ >0 & \text{if } T>T_i^* \end{cases} \quad (24)$$

4.1. Some Theorems

This section is described the useful theorems to present the solution procedure of the optimal inventory cycle time.

Theorem 1:

(1) $\frac{dPV_1(T)}{dT}=0$ has a unique solution T_1^* on $(0,\infty)$.

(2) $\frac{dPV_2(T)}{dT}=0$ has a unique solution T_2^* on $(0,\infty)$.

(3) $\frac{dPV_3(T)}{dT}=0$ has a unique solution T_3^* on $(0,\infty)$.

Proof: The above arguments imply that Theorem 1 holds.

Eq. (24) implies that $PV_i(T)$ is decreasing on $(0,T_i^*]$ and increasing on $[T_i^*,\infty)$ for $i=1,2,3$. Now, consider the case of (A) where $M>\frac{W}{P}$. Eqs. (4), (6) and (8) yield

$PV_1'(W/D) = \frac{e^{-rW/D}}{r(1-e^{-rW/D})^2}\Big[hcDe^{-rW/P}\big(e^{rW/D}-1\big) - hcP\big(1-e^{-rW/P}\big) + cDr\big(e^{rW/D}-1\big) - (A+cW)r^2\Big]$
(25)

and

$PV_2'(W/D) = \frac{e^{-rW/D}}{r(1-e^{-rW/D})^2}\Big[(h+r)cDe^{-rM}\big(e^{rMP/D}-1\big) - (h+r)cP(1-e^{-rM}) - Ar^2\Big]$(26)

We let

$\Delta_1 = hcDe^{-rW/P}\big(e^{rW/D}-1\big) - hcP\big(1-e^{-rW/P}\big) + cDr\big(e^{rW/D}-1\big) - (A+cW)r^2$ (27)

$\Delta_2 = (h+r)cDe^{-rW/P}\big(e^{rW/D}-1\big) - (h+r)cP\big(1-e^{-rW/P}\big) - Ar^2$ (28)

$\Delta_3 = (h+r)cDe^{-rM}\big(e^{rMP/D}-1\big) - (h+r)cP(1-e^{-rM}) - Ar^2$ (29)

Since $PV_2'(T)$ is increasing on $T>0$, we have $PV_2'(MP/D) = PV_2'(W/D)$. So we get that $\Delta_3>\Delta_2$ if $M>W/P$. From Eqs. (25) and (26), we have

$PV_1'(W/D) - PV_2'(W/D) = \frac{e^{-rW/D}}{r(1-e^{-rW/D})^2}\Big[cDr\big(e^{rW/D}-1\big)\big(1-e^{-rW/P}\big) + c\mathrm{Pr}\left(1-e^{-rW/P}\right)\Big] > 0$ (30)

Therefore, $\Delta_1>\Delta_2$. Eqs. (28)-(30) yield

$\Delta_1<0$ if and only if $PV_1'(W/D)<0$ if and only if $T_1^*>\frac{W}{D}$, (31)

$\Delta_2<0$ if and only if $PV_2'(W/D)<0$ if and only if $T_2^*>\frac{W}{D}$ (32)

$\Delta_3<0$ if and only if $PV_2'(MP/D)<0$ if and only if $T_2^*>\frac{MP}{D}$, (33)

$\Delta_3<0$ if and only if $PV_3'\left(\frac{MP}{D}\right)<0$ if and only if $T_3^*>\frac{MP}{D}$. (34)

Hence, we have the following results.

Theorem 2:

(1) If $\Delta_1>0, \Delta_2\geq 0$ and $\Delta_3>0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Hence T^* is T_1^* or W/D associated with the least cost.

(2) If $\Delta_1>0, \Delta_2<0$ and $\Delta_3>0$, then $PV_\infty(T^*) = PV_\infty(T_2^*)$. Hence $T^*=T_2^*$.

(3) If $\Delta_1>0, \Delta_2<0$ and $\Delta_3\leq 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence $T^*=T_3^*$.

(4) If $\Delta_1\leq 0, \Delta_2<0$ and $\Delta_3>0$, then $PV_\infty(T^*) = PV_\infty(T_2^*)$. Hence $T^*=T_2^*$.

(5) If $\Delta_1\leq 0, \Delta_2<0$ and $\Delta_3\leq 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence $T^*=T_3^*$.

Proof: (1) $\Delta_1>0, \Delta_2\geq 0$ and $\Delta_3>0$, which imply that $PV_1'(W/D)>0, PV_2'(W/D)\geq 0$, $PV_2'(MP/D)>0$, and $PV_3'(MP/D)>0$. Eqs. (31)-(34) imply that $T_1^*<W/D, T_2^*\leq W/D$, $T_2^*<MP/D$ and $T_3^*<MP/D$, respectively. Furthermore, Eq. (24) implies that

(i) $PV_3(T)$ is increasing on $[MP/D,\infty)$.

(ii) $PV_2(T)$ is increasing on $[W/D, MP/D)$.

(iii) $PV_1(T)$ is decreasing on $(0,T_1^*]$ and increasing on $[T_1^*, W/D)$.

Combining (i), (ii) and (iii), we conclude that $PV_\infty(T)$ has the minimum value at $T=T_1^*$ on $(0,W/D)$ and $PV_\infty(T)$ has the minimum value at $T=W/D$ on $[W/D,\infty)$. Hence $PV_\infty(T^*) = \min\{PV_\infty(T^*), PV_\infty(W/D)\}$. Consequently, T^* is T_1^* or W/D associated with the least cost.

(2) $\Delta_1>0, \Delta_2<0$ and $\Delta_3>0$, which imply that $PV_1'(W/D)>0, PV_2'(W/D)<0, PV_2'(MP/D)>0$, and $PV_3'(MP/D)>0$. Eqs. (31)-(34) imply that $T_1^*<$

$W/D, T_2^* > W/D, T_2^* < MP/D$ and $T_3^* < MP/D$, respectively. Furthermore, Eq. (24) implies that
(i) $PV_3(T)$ is increasing on $[MP/D, \infty)$.
(ii) $PV_2(T)$ is decreasing on $[W/D, T_2^*]$ and increasing on $[T_2^*, MP/D)$.
(iii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $[T_1^*, W/D)$. Combining (i), (ii) and (iii), we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_2^*$ on $[W/D, \infty)$. Since $PV_1(T) > PV_2(T)$ if $T > 0$, then $PV_\infty(T^*) = PV_\infty(T_2^*)$. Consequently, T^* is T_2^*.
(3) $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 \le 0$, which imply that $PV_1'(W/D) > 0, PV_2'(W/D) < 0, PV_2'(MP/D) \le 0$, and $PV_3'(MP/D) \le 0$. Eqs. (31)-(34) imply that $T_1^* < W/D, T_2^* > W/D, T_2^* \ge MP/D$ and $T_3^* \ge MP/D$, respectively. Furthermore, Eq. (24) implies that
(i) $PV_3(T)$ is increasing on $[MP/D, T_3^*]$ and increasing on $[T_3^*, \infty]$.
(ii) $PV_2(T)$ is decreasing on $[W/D, MP/D]$.
(iii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $[T_1^*, W/D)$.
Combining (i), (ii) and (iii), we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Since $PV_2(T)$ is decreasing on $(0, T_2^*], T_1^* < W/D$ and $T_2^* \ge MP/D > W/D$, we have $PV_1(T_1^*) > PV_2(T_1^*), PV_2(T_1^*) > PV_2(MP/D)$ and $PV_2(MP/D) = PV_3(MP/D) > PV_3(T_3^*)$. Hence, we conclude that $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently, T^* is T_3^*.
(4) $\Delta_1 \le 0, \Delta_2 < 0$ and $\Delta_3 > 0$, which imply that $PV_1'(W/D) \le 0, PV_2'(W/D) \le 0, PV_2'(MP/D) > 0$, and $PV_3'(MP/D) > 0$. Eqs. (31)-(34) imply that $T_1^* \le W/D, T_2^* > W/D, T_2^* < MP/D$ and $T_3^* < MP/D$, respectively. Furthermore, Eq. (24) implies that
(i) $PV_3(T)$ is increasing on $[MP/D, \infty)$.
(ii) $PV_2(T)$ is decreasing on $[W/D, T_2^*]$ and increasing on $T_2^*, MP/D)$.
(iii) $PV_1(T)$ is decreasing on $(0, W/D)$.
Since $PV_1(W/D) > PV_2(W/D)$ and $PV_2'(W/D) > PV_2'(T_2^*)$, combining (i), (ii) and (iii), we conclude that $PV_\infty(T)$ has the minimum value at $T = T_2^*$ on $(0, \infty)$ and consequently, T^* is T_3^*.
(5) $\Delta_1 \le 0, \Delta_2 < 0$ and $\Delta_3 \le 0$, which imply that $PV_1'(W/D) < 0, PV_2'(W/D) < 0, PV_2'(MP/D) < 0$, and $PV_3'(MP/D) < 0$. Eqs. (31)-(34) imply that $T_1^* \ge W/D, T_2^* \ge W/D, T_2^* \ge MP/D$ and $T_3^* \ge MP/D$, respectively. Furthermore, Eq. (24) implies that
(i) $PV_3(T)$ is increasing on $[MP/D, T_3^*]$ and increasing on $[T_3^*, \infty]$.
(ii) $PV_2(T)$ is decreasing on $[W/D, MP/D]$.
(iii) $PV_1(T)$ is decreasing on $(0, W/D)$.
Since $PV_1(W/D) > PV_2(W/D)$, combining (i), (ii) and (iii), we conclude that $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently, T^* is T_3^*.
Combining the above arguments, we have completed the proof of Theorem 2.
In the case of (B) where $M \le W/P, PV_\infty(T)$ can be expressed as follows:

$$PV_\infty(T) = \begin{cases} PV_1(T) & \text{if } 0 < T < W/D \\ PV_3(T) & \text{if } W/D \le T. \end{cases} \quad (35)$$

Eq. (8) yield that

$$PV_3'(W/D) = \frac{e^{-rW/D}}{r(1-e^{-rW/D})^2}\left[cD\left(he^{-rW/P} + re^{-rM}\right)\left(e^{rW/D} - 1\right) - cP\left(h\left(1 - e^{rW/P}\right) + r\left(1 - e^{-rM}\right)\right) - Ar^2 - c(W - PM)r^2e^{-rM}\right]. \quad (36)$$

We let

$$\Delta_4 = cD\left(he^{-rW/P} + re^{-rM}\right)\left(e^{rW/D} - 1\right) - cP\left(h\left(1 - e^{rW/P}\right) + r\left(1 - e^{-rM}\right)\right) - Ar^2 - c(W - PM)r^2e^{-rM} \quad (37)$$

From Eqs. (28)-(37), we have

$$\Delta_1 - \Delta_4 = cDr(1 - e^{-rM})\left(e^{rW/D} - 1\right) + cPr(1 - e^{-rM}) - cWr^2(1 - e^{-rM}) + PMr^2e^{-rM} > 0. \quad (38)$$

From Eq. (37), we also find that
$\Delta_4 < 0$ if and only if $PV_3'(W/D) < 0$ if and only if $T_3^* > W/D$. (39)
Then, we have the following results.

Theorem 3:
(1) If $\Delta_1 > 0$, and $\Delta_4 \ge 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Hence T^* is T_1^* or W/D associated with the least cost.
(2) If $\Delta_1 > 0$, and $\Delta_4 < 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(T_3^*)\}$. Hence T^* is T_1^* or T_3^* associated with the least cost.
(3) If $\Delta_1 \le 0$, and $\Delta_4 < 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence $T^* = T_3^*$.

Proof: Omitted

4.2. The algorithm

In this section, we would combine Section 3.1. to outline the algorithm to help us to decide the optimal replenishment cycle time and optimal production quantity.
The algorithm:
Step 1: If $M \le W/P$, then go to Step 3. Otherwise, go to Step 2. Step 2:
(1) If $\Delta_1 > 0, \Delta_2 \ge 0$ and $\Delta_3 > 0$, then T^* is T_1^* or W/D associated with the least cost.
(2) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 > 0$, then $T^* = T_2^*$.
(3) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 \le 0$, then $T^* = T_3^*$.
(4) If $\Delta_1 \le 0, \Delta_2 < 0$ and $\Delta_3 > 0$, then $T^* = T_2^*$.
(5) If $\Delta_1 \le 0, \Delta_2 < 0$ and $\Delta_3 \le 0$, then $T^* = T_3^*$.
Step 3:
(1) If $\Delta_1 > 0$, and $\Delta_4 \ge 0$, then T^* is T_1^* or W/D associated with the least cost.
(2) If $\Delta_1 > 0$, and $\Delta_4 < 0$, then T^* is T_1^* or T_3^* associated with the least cost.
(3) If $\Delta_1 \le 0$, and $\Delta_4 < 0$, then $T^* = T_3^*$.

5. Numerical examples

To illustrate the results, let us apply the proposed method to solve the following numerical examples. We use the LINGO software to run the algorithm. The following parameters $A = \$5/$ order, $c = \$1$, $D = 15$ units, $P = 20$ units and $h = 0.1$ /unit are used from Examples 18. In addition, $r = 0.3$ is used from Examples 1-6.

Example 1. If $M = 30$ and $W = 100$, then $W/P = 5 < M, \Delta_1 = 19.885 > 0, \Delta_2 = 1.889 > 0$ and $\Delta_3 =$

$112.063 > 0$. Using Step 2(1), we get $T^* = W/D = 6.667$ and $PV_\infty(T^*) = 124.738$.

Example 2. If $M = 30$ and $W = 30$, then $W/P = 1.5 < M, \Delta_1 = 0.611 > 0, \Delta_2 = -0.204 < 0$ and $\Delta_3 = 112.063 > 0$. Using Step 2(2), we get $T^* = T_2^* = 2.745$ and $PV_\infty(T^*) = 65.239$.

Example 3. If $M = 2$ and $W = 30$, then $W/P = 1.5 < M, \Delta_1 = 0.611 > 0, \Delta_2 = -0.204 < 0$ and $\Delta_3 = -0.204 < 0$. Using Step 2(3), we get $T^* = T_3^* = 2.691$ and $PV_\infty(T^*) = 65.242$.

Example 4. If $M = 30$ and $W = 10$, then $W/P = 0.5 < M, \Delta_1 = -0.346 < 0, \Delta_2 = -0.421 < 0$ and $\Delta_3 = 112.063 > 0$. Using Step 2(4), we get $T^* = T_2^* = 2.745$ and $PV_\infty(T^*) = 65.239$.

Example 5. If $M = 2$ and $W = 10$, then $W/P = 0.5 < M, \Delta_1 = -0.346 < 0, \Delta_2 = -0.421 < 0$ and $\Delta_3 = -0.204 < 0$. Using Step 2(5), we get $T^* = T_3^* = 2.691$ and $PV_\infty(T^*) = 65.242$.

Example 6. If $M = 2$ and $W = 50$, then $W/P = 2.5 > M, \Delta_1 = 2.945 > 0$, and $\Delta_4 = 0.755 > 0$. Using Step 3(1), we get $T^* = T_1^* = 1.345$ and $PV_\infty(T^*) = 76.634$.

Example 7. If $r = \$0.1/\$, M = 2$ and $W = 42$, then $W/P = 2.1 > M, \Delta_1 = 0.029 > 0$, and $\Delta_4 = -0.018 < 0$. Using Step 3(2), we get $T^* = T_3^* = 3.14$ and $PV_\infty(T^*) = 180.38$.

Example 8. If $r = \$0.01/\$, M = 2$ and $W = 42$, then $W/P = 2.1 > M, \Delta_1 = -0.00029 < 0$, and $\Delta_4 = -0.00034 < 0$. Using Step 3(3), we get $T^* = T_3^* = 4.549$ and $PV_\infty(T^*) = 1710.269$.

6. Conclusion

This paper presents the discounted cash-flows (DCF) approach for the analysis of the optimal inventory policy in the presence of trade credit depending on the producing quantity. If $Q < W$, the delay in payment is not permitted. Otherwise, the fixed trade period M is permitted. There are two cases (1) $M > W/P$ and (2) $M \leq W/P$ to be explored. Theorem 2 gives the solution procedure to find T^* when $M > W/P$. Theorem 3 gives the solution procedure to find T^* when $M \leq W/P$. Numerical examples are given to illustrate Theorems 2 and 3. Furthermore, an algorithm to find the optimal replenishment cycle time is presented.

References

[1] Huang Y.F., Chou C.L., Liao J.J., An EPQ model under cash discount and permissible delay in payments derived without derivatives, Yugoslav Journal of Operations Research, 2007, Vol. 17, No. 2, pp. 177193.

[2] Teng J.-T., On the economic order quantity under conditions of permissible delay in payments, Journal of the Operational Research Society, 2002, Vol. 53, No. 8, pp. 915918.

[3] Chung K., Huang Y., The optimal cycle time for EPQ inventory model under permissible delay in payments, International Journal of Production Economics, 2003, Vol. 84, No. 3, pp. 307318.

[4] Huang Y.F., Chung K.J., Optimal replenishment and payment policies in the EOQ model under cash discount and trade credit, Asia-Pacific Journal of Operational Research, 2003, Vol. 20, No. 2, pp. 177190.

[5] Huang Y.F., Huang H.F., Optimal inventory replenishment policy for the EPQ model under trade credit derived without derivatives, International Journal of Systems Science, 2008, Vol. 39, No. 5, pp. 539546.

[6] Chung K., Huang Y., The optimal cycle time for EPQ inventory model under permissible delay in payments, International Journal of Production Economics, 2003, Vol. 84, No. 3, pp. 307318.

[7] Hu F., Liu D., Optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages, Applied Mathematical Modelling, 2010, Vol. 34, No. 10, pp. 31083117.

[8] Chung K.J., Huang C.K., An ordering policy with allowable shortage and permissible delay in payments, Applied Mathematical Modelling, 2009, Vol. 33, No. 5, pp. 25182525.

[9] Hou K.L., Lin L.C., Optimal pricing and ordering policies for deteriorating items with multivariate demand under trade credit and inflation, Opsearch, 2013, Vol. 50, No. 3, pp. 404417.

[10] Chung K.J., The EPQ model under conditions of two levels of trade credit and limited storage capacity in supply chain management, International Journal of Systems Science, 2013, Vol. 44, No. 9, pp. 16751691.

[11] Huang Y.F., Optimal retailer's ordering policies in the EOQ model under trade credit financing, Journal of the Operational Research Society, 2003, Vol. 54, No. 9, pp. 10111015.

[12] Khouja, M., Mehrez, A., 1996. Optimal inventory policy under different supplier credits. Journal of Manufacturing Systems 15, 334339

[13] Molamohamadi Z., Arshizadeh R., Ismail N., An EPQ inventory model with allowable shortages for deteriorating items under trade credit policy, Discrete Dynamics in Nature and Society, 2014, vol. 2014, http://dx.doi.org/10.1155/2014/476085.

[14] Chung, K.J., Liao, J.J., The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach, European Journal of Operational Research, 2009, Vol. 196, No. 2, 563568.

[15] Chung, K. H., Inventory control and trade credit revisited, Journal of the Operational Research Society, 1989, Vol. 40, No. 5, pp. 495498.