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Reliability optimization of fully fuzzy redundancy allocation problem in uncertain environment via soft computing technique

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Abstract

Reliability optimization of a redundancy allocation problem is an important area of research in the field of literature. The main goal of these problems is to enhance the reliability of the system. In this paper, we have considered *n*-stage series system with redundant units in parallel with multiple constraints both in crisp and intuitionistic fuzzy environments. Thus, the crisp non-linear integer programming problem has been formulated as fuzzy intuitionistic non-linear integer programming models. For the ease of solution of the problem, this has been crispified using the Beta distribution method. Then the problem has been solved using real coded elitist genetic algorithm for integer variables. The results obtained by these methods along with that for crisp model are compared and are presented in tabular form. Finally, a numerical example is solved to clarify the sensitivity of the proposed GA with respect to the crisp and intuitionistic fuzzy environments.

Keywords: Redundancy allocation problem (RAP), Genetic algorithm, Trapezoidal intuitionistic fuzzy number, Beta distribution method of defuzzification, n-stage series system.

1. Introduction

The theory of reliability has been introduced to fulfill the demands of modern technology, especially due to experiences gained from complex military systems in World War II. Most of the problems attempted earlier were machine maintenance and system reliability in military systems. Today reliability is considered in almost all engineering system designs. Problem of the reliability in various designs of system have been explored by Tillman et al. [18], Misra [14], Kuo et al. [9] and so many researchers. We are trying to raise the reliability of those equipment or systems as high as possible, because of imperfectness of these systems or components due to technical defects could lead to serious damages including loss of life and property.

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Redundancy allocation is useful in reliability optimization of a system. The main goal of this problem is to add component(s) to the system so as to maximize the system reliability subject to the constraints on the system/subsystem.

Many reliability engineers/researchers have indulged themselves for solving the reliability-redundancy allocation problem (RAP) which mainly appears as a nonlinear integer/mixed integer programming problem. It has been studied and summarized in [15, 16, 17, 27, 28, 33, 36, 37] and it is also NP- hard. Also, different types of reliability optimization problems in different environments are discussed in [38-45].

It is quite mandate to enhance the reliability of a system as more and more. There are so many objectives for research on the redundancy allocation problem. Reliability maximization was the first and utmost purpose of these researches. The decision variables or the design parameters in redundancy allocation problem may be the component reliability value and/or the number of arrangements of the known components. Using particle swarm optimization method, Khalili-Damghani [8] solved reliability redundancy problem. Dolatshahi-Zand and Khalili-Dhamaghani [5], designed the supervisory control and data acquisition (SCADA) water resource management control center using a bi-objective redundancy allocation problem and particle swarm optimization.

For solving such redundancy allocation problem, several deterministic methods, like heuristic methods [26, 33], reduced gradient method [25], linear programming approach [26], dynamic programming method [28], branch and bound method [35] were used in the initial stage of development. Later, the evolutionary algorithms were being used for solving RAP. Evolutionary algorithms provide more flexibility; require less assumption on the objective as well as on the constraints. These algorithms work well for both the discrete and continuous search space.

In the existing literature, most of the studies have considered the design parameters to be precise

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valued. This means that every probability involved is perfectly determinable and there exist complete information about the system and the component behavior. However, in real life situations, there are not sufficient statistical data available in most of the cases where either the system is new or if exists only as a project. It is not always possible to observe the stability from the statistical point of view. This means that only some partial information about the system components is known. So, the reliability of a component of a system will be an imprecise number which can be represented by different approaches like fuzzy, intuitionistic fuzzy, stochastic and interval approaches or combination of these.

Several works in reliability redundancy optimization in fuzzy environment have been encountered in literature. Huang [7] has presented the optimization of a series system in fuzzy environment considering the concept of multi-objective decision making. Mahapatra and Roy [11] have also used the fuzzy multi objective technique on reliability optimization. They [12] have extended their work for reliability evaluation using intuitionistic fuzzy numbers. They [13] have presented the optimal redundancy allocation using generalized fuzzy number in a series- parallel system. They also [29] have considered a bridge system with fuzzy reliability of the components using interval nonlinear programming. Mahato et al. [31] have discussed fuzzy reliability redundancy optimization with signed distance method for defuzzification.

Though several works are available in different environments like fuzzy, interval and stochastic for reliability optimization, only few researchers have attempted the reliability redundancy optimization in intuitionistic fuzzy environment. The trapezoidal intuitionistic fuzzy numbers have been considered in reliability optimization through the arithmetic operations of the TrIFN [12]. Here, we have developed some new techniques of crispification of fuzzy numbers.

In this paper, we have discussed the optimization of system reliability for n-stage series system with redundant units in parallel. The corresponding problem has been formulated in crisp and intuitionistic fuzzy environments. We have used average beta distribution technique as a crispification method after extension of them for intuitionistic fuzzy numbers. Then these problems have been formulated as unconstrained integer programming problems using the Big-M penalty technique. The reduced problems have been solved by the real coded elitist genetic algorithm (RCEGA) for integer variables, tournament selection, intermediate crossover and one neighborhood mutation. The comparative study of the results so obtained has been presented and also the sensitivity studies are carried out and presented in tabular form and also graphically.

The remaining part of this work is framed as follows: In section 2, the basic assumptions and symbols that are used. The definitions of fuzzy, trapezoidal fuzzy and trapezoidal intuitionistic fuzzy numbers with graphical representation are given in section 3. Section 4 provides the crispification techniques for the considered intuitionistic fuzzy number. The formulation of the problem in crisp and intuitionistic fuzzy environments is kept in section 5. The solution methodology is described in sections 6. Section 7 describes the proposed soft computing techniques. For the clarification of our proposed environment and algorithm, numerical example is taken and sensitivities are drawn graphically in section 8. The obtained results of this study are analyzed with the inclusion of section 9. Section 10 concludes the whole work done in this paper with some future scopes.

2. Assumptions and notation

2.1 Assumptions

In the entire work, we have taken the following assumptions:

- n-stage series system is taken.
- The cost and weight coefficients are trapezoidal intuitionistic fuzzy valued.
- The components reliabilities are precise/ intuitionistic fuzzy valued.
- Redundant components are active and non-repairable.
- Each subsystem is comprised of identical components.
- System reliability is not dependent on the failure of components of the subsystems.

2.2 Notation

Symbols	Meanings
n	subsystems' number
x_{ij}	number of redundant components of design alternative j in stage i
T_{ij}	reliability of design alternative j in stage i

$\tilde{T}_{ij}$	intuitionistic fuzzy reliability of design alternative *j* in stage *i*
$T_S(x), \tilde{T}_S(x)$	crisp, intuitionistic fuzzy system reliability (objective function)
c_{ij}	cost of the design alternative j in stage i
$\tilde{c}_{ij}$	intuitionistic fuzzy cost of the design alternative j in stage i
w_{ij}	weight of the design alternative j in stage i
$\widetilde{w}_{ij}$	intuitionistic fuzzy weight of the design alternative j in stage i
C_0	cost boundary
$\tilde{C}_0$	intuitionistic fuzzy cost boundary
W_0	weight boundary
$\widetilde{W}_0$	intuitionistic fuzzy weight boundary
S	feasible region
l_i	number of components in the stage i
l_{ij}	lower bound of redundant components of design alternative j in stage i
u_{ij}	upper bound of redundant components of design alternative j in stage i

3. Some preliminaries

3.1 Fuzzy Number: A fuzzy number $(\hat{A})$ is a fuzzy set which is both convex and normal. That is a fuzzy number is a special case of a fuzzy set. The pictorial representation of a fuzzy number is given in Figure 1.

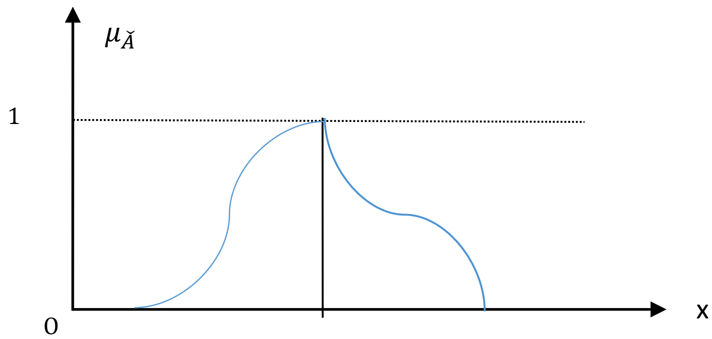

Figure 1: Membership function of $\hat{A}$

3.2 Trapezoidal Fuzzy Number (TrFN)

A trapezoidal fuzzy number (TrFN) $\tilde{A}$ is a fuzzy set in R with following membership function $(\mu_{\tilde{A}}(x))$

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, \text{for } a_1 \le x \le a_2 \\ 1, \quad for\ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, \text{for } a_3 \le x \le a_4 \\ 0, \text{otherwise} \end{cases}$$

where, $a_1 \le a_2 \le a_3 \le a_4 \ \ \forall x \in R$

This TrFN is denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$

A trapezoidal fuzzy number is shown pictorially in Figure 2.

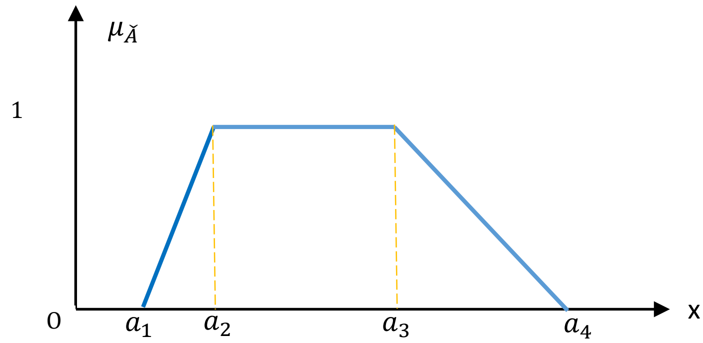

Figure 2: Trapezoidal fuzzy number

3.3. Trapezoidal Intuitionistic fuzzy number (TrIFN)

A trapezoidal intuitionistic fuzzy number $(\hat{A})$ is an intuitionistic fuzzy number whose respective membership and non-membership functions are given by

$$\mu_{\hat{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, \text{for } a_1 \le x \le a_2 \\ 1, \quad for\ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, \text{for } a_3 \le x \le a_4 \\ 0, \text{otherwise} \end{cases}$$

and $\nu_{\hat{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, \text{ for } a_1' \le x \le a_2 \\ 0, \quad for\ a_2 \le x \le a_3 \\ \frac{x-a_3}{a_4'-a_3}, \text{for } a_3 \le x \le a_4' \\ 1, \text{ otherwise.} \end{cases}$

where, $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$ and for $\mu_{\hat{A}}(x) = \nu_{\hat{A}}(x),\ \mu_{\hat{A}}(x) \& \nu_{\hat{A}}(x) \le 0.5\ \forall -\infty \le x \le \infty$.

In general, a TrIFN is represented by $\hat{A} = (a_1, a_2, a_3, a_4\ ; a_1', a_2, a_3, a_4')$. The pictorial view of TrIFN is depicted in Figure 3.

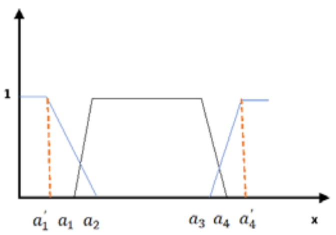

Figure 3: Trapezoidal intuitionistic fuzzy number

4. Beta distribution method of defuzzification

According to Rahmani et al. (47), the defuzzification of trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is given as $\beta_{\tilde{A}} = \frac{(2a_1+7a_2+7a_3+2a_4)}{18}$.

4.1 Average beta distribution method of defuzzification

For the trapezoidal intuitionistic fuzzy number $\hat{A} = (a_1, a_2, a_3, a_4\ ; a_1', a_2, a_3, a_4')$ the corresponding defuzzification formula of membership and non-membership functions are as follows:

$\beta_{\widehat{A_\mu}} = \frac{(2a_1+7a_2+7a_3+2a_4)}{18}$ and $\beta_{\widehat{A_\vartheta}} = \frac{(2a_1'+7a_2+7a_3+2a_4')}{18}$

Therefore, taking the mean of $\beta_{\widehat{A_\mu}}$ and $\beta_{\widehat{A_\vartheta}}$, the formula of average beta distribution method of defuzzification of TrIFN $(\hat{A})$ becomes as

$avg(\beta_{\widehat{A_\mu}}) = \frac{(2a_1'+2a_1+14a_2+14a_3+2a_4+2a_4')}{36}$.

5. Formulation of the problem

5.1 The Crisp Model

Let us consider a system reliability of n-stage series system with redundant units in parallel as shown in Figure 4. It is also assumed that in each stage, different types of components can be used as design alternatives. Now, the nonlinear integer programming problem can be formulated as:

$$Maximize\ T_S = \prod_{i=1}^{n}[1-\prod_{j=1}^{l_i}(1-T_{ij})^{x_{ij}}] \tag{1}$$

$$\text{Subject to } \sum_{i=1}^{n}\sum_{j=1}^{l_i} c_{ij}x_{ij} \le C_0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{l_i} w_{ij}x_{ij} \le W_0$$

$x_{ij} \ge 0$ and are integers, $1 \le i \le n, 1 \le j \le l_i$.

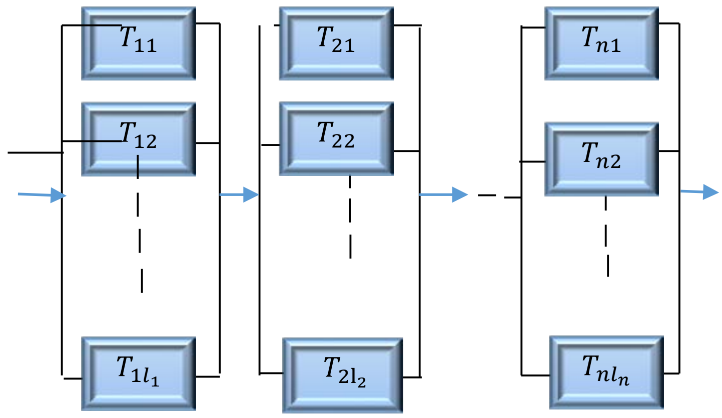

Fig. 4: n-stage series system

5.2 Intuitionistic Fuzzy Model

After considering the cost, weight coefficients and reliability components as trapezoidal intuitionistic fuzzy number problem (1) can be reformulated as fully fuzzy intuitionistic nonlinear integer programming problem (FFINIPP) as follows:

$$Maximize\ \hat{T}_S = \prod_{i=1}^{n}[1-\prod_{j=1}^{l_i}(1-(T_{ij}^{\ 3}, T_{ij}^{\ 1}, T_{ij}^{\ 2}, T_{ij}^{\ 4}; T_{ij}^{\ 5}, T_{ij}^{\ 1}, T_{ij}^{\ 2}, T_{ij}^{\ 6}))^{x_{ij}}] \tag{2}$$

subject to

$$\sum_{i=1}^{n}\sum_{j=1}^{l_i}(c_{ij}^{\ 3}, c_{ij}^{\ 1}, c_{ij}^{\ 2}, c_{ij}^{\ 4}; c_{ij}^{\ 5}, c_{ij}^{\ 1}, c_{ij}^{\ 2}, c_{ij}^{\ 6})x_{ij} \le (C_0^{\ 3}, C_0^{\ 1}, C_0^{\ 2}, C_0^{\ 4}; C_0^{\ 5}, C_0^{\ 1}, C_0^{\ 2}, C_0^{\ 6})$$

$$\sum_{i=1}^{n}\sum_{j=1}^{l_i}(w_{ij}^{\ 3}, w_{ij}^{\ 1}, w_{ij}^{\ 2}, w_{ij}^{\ 4}; w_{ij}^{\ 5}, w_{ij}^{\ 1}, w_{ij}^{\ 2}, w_{ij}^{\ 6})x_{ij} \le (W_0^{\ 3}, W_0^{\ 1}, W_0^{\ 2}, W_0^{\ 4}; W_0^{\ 5}, W_0^{\ 1}, W_0^{\ 2}, W_0^{\ 6})$$

$x_{ij} \ge 0$ and x_{ij} are integers, $1 \le i \le n, 1 \le j \le l_i$.

6. Solution methodology

As the problems (1) and (2) are constrained optimization problems, one can use the solution techniques available in the literature. In this work, we have used the penalty function technique in which the constrained optimization problem is converted into unconstrained optimization problem. The easiest and the most effective Big-M penalty technique [24, 32, 34] has been used in this work. Hence, the unconstrained optimization problems corresponding to the problems (1) and (2) are as follows:

Maximize $$T_s(x) = \begin{cases} T_s(x) & \text{when } x \in S_1 \\ -M & \text{when } x \notin S_1 \end{cases}$$ (3)

where,

$$S_1 = \{x : \sum_{i=1}^{n}\sum_{j=1}^{l_i} c_{ij}x_{ij} \le C_0,\ \sum_{i=1}^{n}\sum_{j=1}^{l_i} w_{ij}x_{ij} \le W_0,\ 1 \le i \le n, 1 \le j \le l_i$$
$$\text{and } 1 \le l_{ij} \le x_{ij} \le u_{ij},\quad x_{ij} \in Z^+\}.$$

Maximize $$\hat{T}_s(x) = \begin{cases} \hat{T}_s(x) & \text{when } x \in S_2 \\ -M & \text{when } x \notin S_2 \end{cases}$$ (4)

where,

$$S_2 = \{x : \sum_{i=1}^{n}\sum_{j=1}^{l_i} \hat{c}_{ij}x_{ij} \le \hat{C}_0,\ \sum_{i=1}^{n}\sum_{j=1}^{l_i} \hat{w}_{ij}x_{ij} \le \hat{W}_0,\ 1 \le i \le n, 1 \le j \le l_i$$
$$\text{and } 1 \le l_{ij} \le x_{ij} \le u_{ij},\quad x_{ij} \in Z^+\}.$$

There are several evolutionary algorithms to solve this problem. In this work, we have developed real coded elitist genetic algorithm for solving the above-mentioned problem.

7. Genetic Algorithm

We have used Genetic Algorithm (GA) in solving the reliability optimization problems described in this paper. GA is a stochastic search and optimization technique based on the evolutionary principle "survival of the fittest" and natural genetics [22, 26, 31, 32, 48,46,]. Gen and Cheng (1997) described the applications of GA to combinatorial problems including reliability optimization problems. The GA has the following popular features:

(i) GA works with a coding of solution set, not the solution themselves,

(ii) GA searches over a population of solutions, not a single solution,

(iii) GA uses payoff information, not derivatives or another auxiliary knowledge,

(iv) GA applies stochastic transformation rules, not deterministic.

The procedural algorithm of the working principle of GA is as follows:

Algorithm

Step-1: Set population size (pop_{size}), crossover probability ($prob_{cross}$), mutation probability ($prob_{mute}$), maximum generation (max_{gen}) and bounds of the variables x_{ij}.

Step-2: $t = 0$ [t represents the number of current generations]

Step-3: Initialize the chromosome of the population $T(t)$ [$T(t)$ represents the population at t-th generation]

Step-4: Evaluate the fitness function of each chromosome of $T(t)$ considering the objective function as the fitness function

Step-5: Find the best chromosome from the population $T(t)$

Step-6: Set t=t+1

Step-7: If the termination criterion is satisfied go to Step-14, otherwise, go to next step

Step-8: Select the population $T(t)$ from the population $T(t-1)$ of earlier generation by tournament selection process.

Step-9: Alter the population $T(t)$ by crossover, mutation and elitism operators

Step-10: Evaluate the fitness function value of each chromosome of $T(t)$

Step-11: Find the best chromosome from $T(t)$

Step-12: Compare the best chromosome of $T(t)$ and $T(t-1)$ and remember the preferred one

Step-13: Go to Step-6

Step-14: Print the best chromosome (which is the solution of the optimization problem)

Step-15: End

In our work, the value of objective function of the reduced unconstrained optimization problems corresponding to the chromosome is considered as the fitness value of that chromosome. We have used tournament selection with size the two, intermediate crossover for integer variables, one-neighborhood mutation for integer variables and the termination condition as the maximum number of generations.

8. Numerical example

For illustration of the proposed problem and solution technique, we have considered the numerical example from Chern and Jan [4]. In this problem, we have considered a 3- stage series system with redundant units in parallel (1-out-of-3: G stage) and assumed that in each stage, different types of components can be used as design alternatives. Here, the system budget and system weight for the crisp model are C_0 =30 and W_0=17 and for the intuitionistic fuzzy model are $\widetilde{C_0} = (29.15,30,31.25,32;\, 28.99,30,31.25,33.19)$ and $\widetilde{W_0} = (16,17,18,19;\, 15.8,17,18,20)$. The complete data required to formulate the problem are given in the tables 1 and 2. Also, the obtained results are kept in Table 3.

Table 1: Input data for crisp problem

j	Items	i		
		1	2	3
1	<u>C</u>	4	8	11
	<u>W</u>	2	3	4
	<u>R</u>	0.99	0.98	0.98
2	C	13	3	5
	W	3	3	6
	R	0.95	0.8	0.92
3	C	7	3	
	W	5	9	
	R	0.92	0.90	
l_i		3	3	2

C_0 =30, W_0=17

Table 2: Input data for FFINIPP

j	Items	i		
		1	2	3
1	$\underline{\tilde{C}}$	(2,4,5,6;1,4,5,7)	(6,8,9,10;5,8,10,12)	(9,11,12,13;8,11,12,15)
	$\underline{\widetilde{W}}$	(1,2,3,4;0.95,2,3,4.7)	(2,3,4,5;1,3,4,6)	(3,4,5,6;2,4,5,7)
	$\underline{\tilde{T}}$	(0.94,096,0.97,0.98; 0.90,0.96,0.97,0.99)	(0.94,096, 0.97,0.98; 0.90,0.96, 0.97,0.99)	(0.86,0.93, 0.95,0.97; 0.83,0.93, 0.95,0.99)
2	$\underline{\tilde{C}}$	(11,13,14,15; 10,13,14,16)	(1,2,4,5;0.5,2,4,6)	(3,5,6,7;2,5,6,9)
	$\underline{\widetilde{W}}$	(2,3,5,6;1,3,5,8)	(2,3,6,8;1,3,6,9)	(5,6,8,9;4,6,8,11)
	$\underline{\tilde{T}}$	(0.85,0.95, 0.96,0.97; 0.80,0.95,0.96,0.99)	(0.75,0.80, 0.83,0.85; 0.70,0.80, 0.83,0.89)	(0.83,0.92, 0.94,0.96; 0.80,0.92, 0.94,0.99)
3	$\underline{\tilde{C}}$	(5,7,8,9;4,7,8,10)	(1,3,4,5;0.5,3,4,5.5)	---------
	$\underline{\widetilde{W}}$	(4,5,7,8;3,5,7,9)	(8,9,11,12;7,9,11),13	
	$\underline{\tilde{T}}$	(0.86,0.92, 0.93,0.95; 0.80,0.92,0.93,0.97)	(0.84,0.90,0.94,0.96; 0.80,0.90, 0.94,0.99)	
l_i		3	3	2

$$\widetilde{C_0} = (29.15,30,31.25,32;\, 28.99,30,31.25,33.19),\ \widetilde{W_0} = (16,17,18,19;\, 15.8,17,18,20)$$

Table 3: Optimal solutions

Environments	*Redundant vector*	T_s
Crisp	(1,0,0,1,0,0,0,2)	0.96399087
Trapezoidal intuitionistic fuzzy	**(3,0,1,1,0,0,1,0)**	**0.99598239**

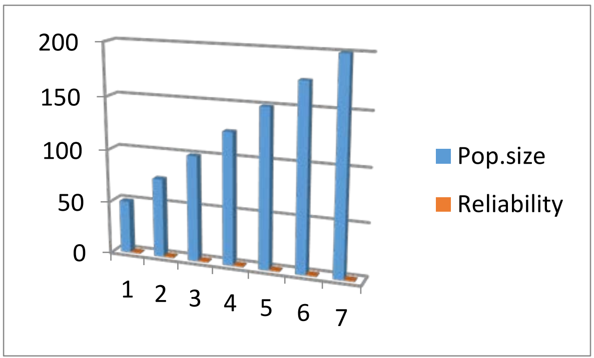

Figure 5: Reliability Vs. Population size

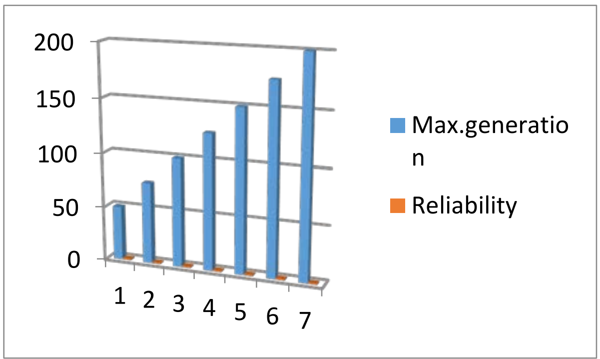

Figure 6: Reliability Vs. Maximum number of generations

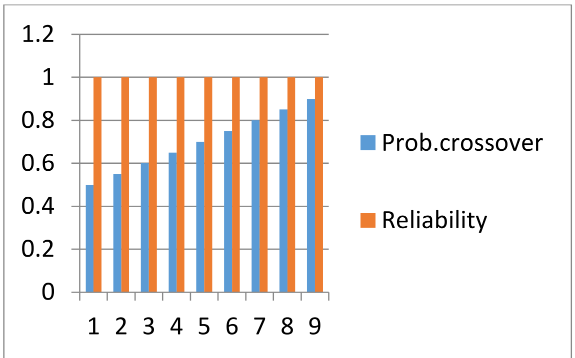

Figure 7: Reliability Vs. Probability of crossover

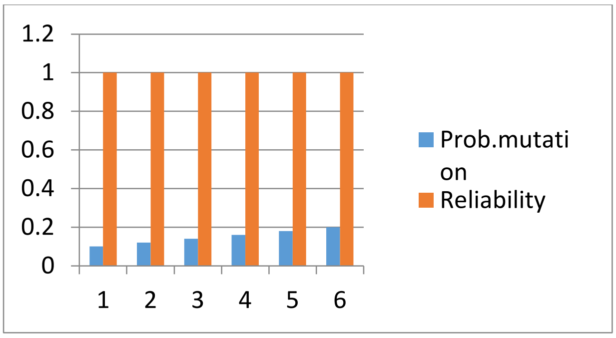

Figure 8: Reliability Vs. Probability of mutation

9. Result analysis

We have considered a numerical example to clarify our strategy for optimizing the system reliability with appropriate redundancy component allocations. We have run the proposed GA 30 times independently in C++ on a notebook with an Intel CORE i3 10th generation CPU and 4GB RAM in WINDOWS. For the considered example, the population size and maximum number of generations taken in the RCEGA are 85 and 200 respectively. The comparative results in crisp and intuitionistic fuzzy environments have been provided in Tables 3. From this table it is clear that the system reliability in trapezoidal intuitionistic fuzzy environment is higher than the crisp environment. The Figures 5-8 were drawn to visualize the behavior of the system reliability with respect to GA parameters in different intuitionistic fuzzy environment. From the figures 5 and 6, it is clear that the system reliability increases as the number of generations and the population size increase. Also, it is clear from the figures 7 and 8 that the system reliability takes higher value with the increment of the probability of crossover and probability of mutation.

10. Conclusions and future scopes

In this paper, an n-stage series system with redundancy in parallel is formulated as no-linear integer programming problem (NIPP) in crisp and trapezoidal intuitionistic fuzzy environments. Then the intuitionistic fuzzy model is transformed after crispifying the TrIFN parametric values by using the average beta distribution method. The Big-M technique is used to convert the constrained optimization problem into unconstrained one. The soft computing technique of GA was used to solve the numerical problem in diverse techniques and forms, including crisp and intuitionistic fuzzy. Our main goal was to obtain the maximum system reliability subject to different resource limitations. Throughout this work we have seen that the intuitionistic fuzzy environment produces a remarkable result for the considered example.

For further study, one may consider other imprecise environments like neutroshophic, type-2 fuzzy etc. for finding maximum of differently designed reliability systems. Also, the researchers have a platform to implement different types of heuristic techniques such as ABC, PSO, QPSO etc. for solving the similar types of problems as considered in this work.

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