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On generalized soft metric spaces

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Abstract Firstly, we have introduced a generalized concept of metric spaces, named generalized soft metric space, based on soft points of soft sets, and some basic properties regarding generalized soft metric space are studied with examples. After that, we established a fixed point theorem on generalized soft metric space.

Keywords: Soft set, Soft point, Generalized soft metric space, Fixed soft point.

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1. Introduction

Soft set is a different approach to managing uncertainty that adheres to real life conditions. It was first introduced by Molodtsov [10] in 1999 as an expansion of fuzzy set theory [19].

Maji et al. [11] then conducted a thorough analysis of this idea. Consequently, researchers began to formalize various mathematical structures in this context, such as metric spaces, normed linear spaces, topology etc [14, 15, 16, 20], and the work advanced quickly.

Majumdar and Samanta [12] introduced the concept of soft mapping. In [14, 15], Das and Samanta presented the ideas of soft real set, soft real number, soft point, and soft metric spaces and examined some of their properties. After that, work in this area is progressing very fast.

Nowadays, Guler et al. [2] have studied the behavior of G-metric spaces in soft set settings and gave the notion of soft G-metric spaces.

Afterwards, Guler and Yildirim [3] have also introduced soft G-complete metric spaces, and some fixed point results are investigated. In 2018, Aras et al. [7, 8] induced soft S-metric spaces, sequential compact soft S-metric spaces, soft complete S-metric spaces, etc. and some of its important properties are discussed.

In 2020, Aras et al. [9] induced soft D-metric spaces. In 2022, Sk. Nazmul and U. Badyakar [17] introduced the concept of soft R-metric spaces and established some fixed point results in this space. In 2024, Sk. Nazmul and U. Badyakar [18] introduced the concept of soft S_b-metric spaces and established some fixed point results in this space.

M. Jleli et al. [21] and C. Vijender [6] introduced the idea of generalized metric spaces. After that, they proved some fixed point results in this space.

In this study, we have introduced the concept of soft generalized metric spaces using soft points. After that, some properties are established with proper justifications. Finally, a fixed point result is established in this space.

1. Preliminaries

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Following [1, 4, 5, 10, 11, 13, 14, 15], we provide certain definitions, which are essential for the main discussions.

Definition 2.1 *Let $\mathcal{F}: A \to P(X)$ be a mapping, where $P(X)$ be the power set of a set*

$X\ (\neq \phi)$ and $A\ (\neq \phi) \subseteq E$, the set of parameters. Then $(\mathcal{F}, A)$ is named a soft set over X.

Definition 2.2 *The soft set $(\mathcal{F}, A)$ over X, is said to be*

• an absolute soft set if $\mathcal{F}(a) = X, \forall\ a \in A$ and it is denoted by $\tilde{X}$.

• a soft point if $\mathcal{F}(a) \in X$ and $\mathcal{F}(b) = \phi, \forall\ b \neq a$. If $\mathcal{F}(a) = \{x\}$, then the corresponding soft point is denoted by P_a^x.

Definition 2.3 *A soft real set is a mapping $\mathcal{F}: A \to \mathcal{B}(\mathbb{R})$, where $\mathcal{B}(\mathbb{R})$ is the collection of all non-empty bounded subsets of $\mathbb{R}$, the set of all real numbers.*

A soft real set is called soft real number if $\mathcal{F}(a) \in \mathbb{R}$. $\tilde{r}$ is denoted by a soft real number where $\tilde{r}(a)$ is an element of $\mathbb{R}, \forall\ a \in A$ and $\overline{r}$ is a special type of soft real number where $\overline{r}(a) = r, \forall\ a \in A$.

Definition 2.4 *Two soft points P_a^x and P_b^y in $\tilde{X}$ are said to be unequal if either $x \neq y$ or $a \neq b$.*

Definition 2.5 *For any soft real numbers $\tilde{r}, \tilde{s}$ in $(\mathbb{R}, A)$, we say $\tilde{r}\ \tilde{\leqq}\ (\tilde{<})\ \tilde{s}$ or equivalently $\tilde{s}\ \tilde{\geqq}\ (\tilde{>})\ \tilde{r}$ if $\tilde{r}(a) \leq (<) \tilde{s}(a), \forall\ a \in A$.*

Definition 2.6 *Let $S(X, A)$ and $S(Y, B)$ be the families of all soft sets over X and Y respectively. The mapping $f_\varphi: S(X, A) \to S(Y, B)$ is called soft mapping from X to Y; where $f: X \to Y$ and $\varphi: A \to B$ are two mappings such that the image of a soft set $(\mathcal{F}, A)\ \tilde{\in}\ S(X, A)$ under the mapping f_φ is denoted by $f_\varphi(\mathcal{F}, A) = (f_\varphi(\mathcal{F}), B)$ and is defined by,*

$$[f_\varphi(\mathcal{F})](\beta) = \begin{cases} \bigcup_{\alpha \in \varphi^{-1}(\beta)} [f(\mathcal{F}(\alpha))], & if \quad \varphi^{-1}(\beta) \neq \phi \\ \phi, & otherwise\ . \end{cases}$$

Definition 2.7 *Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$ and $\mathbb{R}(A)^*$ be the set of all non-negative soft real numbers. A mapping $\tilde{\rho}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A)^*$, satisfying the following conditions, $\forall\ P_{\lambda_1^*}^x, P_{\lambda_2^*}^y, P_{\lambda_3^*}^z\ \tilde{\in}\ SP(\tilde{X})$ and $\overline{s}\ \tilde{\geqq}\ \overline{1}$,*

• $\tilde{\rho}(P_{\lambda_1^*}^x, P_{\lambda_2^*}^y)\ \tilde{\geqq}\ \overline{0}$,

• $\tilde{\rho}\left(P_{\lambda_1^*}^x, P_{\lambda_2^*}^y\right) \cong \overline{0}$ *if and only if* $P_{\lambda_1^*}^x\ \cong\ P_{\lambda_2^*}^y$,

• $\tilde{\rho}(P_{\lambda_1^*}^x, P_{\lambda_2^*}^y)\ \cong\ \tilde{\rho}(P_{\lambda_2^*}^y, P_{\lambda_1^*}^x)$,

• $\tilde{\rho}(P_{\lambda_1^*}^x, P_{\lambda_2^*}^y)\ \tilde{\leqq}\ \overline{s}\ \{\tilde{\rho}(P_{\lambda_1^*}^x, P_{\lambda_3^*}^z) + \tilde{\rho}(P_{\lambda_3^*}^z, P_{\lambda_2^*}^y)\}$,

is called a soft b -metric on $\tilde{X}$ and $(\tilde{X}, \tilde{\rho}, A)$ is a soft b -Metric Space.

3. Generalized Soft Metric Spaces

Let X be a nonempty set, A be a nonempty set of parameters and

$\mathcal{D}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A)^*$ be a given mapping. For every $P_\lambda^x \in \tilde{X}$, let us define the set

$$C(\mathcal{D}, \tilde{X}, P_\lambda^x)\ \cong\ \{\{P_{\lambda,n}^x\} \subset \tilde{X}: \lim_{n\to\infty} \mathcal{D}(P_{\lambda,n}^x, P_\lambda^x)\ \cong\ \overline{0}\}.$$

Definition 3.1 *We say that $\mathcal{D}$ is Soft generalized metric on $\tilde{X}$ if it satisfies the following conditions:*

• $\mathcal{D}(P_\lambda^x, P_\mu^y)\ \tilde{\geqq}\ \overline{0}, \forall\ P_\lambda^x, P_\mu^y \in \tilde{X}$.

• for every $P_\lambda^x, P_\mu^y\ \tilde{\in}\ \tilde{X}$, we have

$$\mathcal{D}(P_\lambda^x, P_\mu^y)\ \cong\ \overline{0} \Rightarrow P_\lambda^x\ \cong\ P_\mu^y$$

- for every $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$, we have

$$\mathcal{D}(P_\lambda^x, P_\mu^y) \cong \mathcal{D}(P_\mu^y, P_\lambda^x)$$

- there exist $\tilde{C} \tilde{>} \overline{1}$ such that if $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$, $\{P_{\lambda,n}^x\} \tilde{\in} C(\mathcal{D}, \tilde{X}, P_\lambda^x)$, then $\mathcal{D}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{C} \lim\sup_{n\to\infty} \mathcal{D}(P_{\lambda,n}^x, P_\mu^y)$.

In this case, we say the pair $(\tilde{X}, \mathcal{D})$ is a generalized soft metric space.

Example 3.2 *Let $X(\neq \phi) \subset \mathbb{R}$ and $A(\neq \phi)$ be finite subset of $\mathbb{R}$.*

Define $\mathcal{D}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A)^*$ by $\mathcal{D}(P_\lambda^x, P_\mu^y) \cong |\overline{x} - \overline{y}| + |\overline{\lambda} - \overline{\mu}|, \forall\, P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$.

Choose $\{P_{\lambda,n}^x\}$, sequence of soft points of (X, A), such that $P_{\lambda,n}^x(\lambda) = \frac{1}{n}, \forall\, n \in \mathbb{N}$.

Then clearly $(\tilde{X}, \mathcal{D})$ is a generalized soft metric space.

Remark 3.3 *If the set $C(\mathcal{D}, \tilde{X}, P_\lambda^x)$ is empty for every $P_\lambda^x \in \tilde{X}$, then $(\tilde{X}, \mathcal{D})$ is a generalized soft metric space if and only if (1) and (2) are satisfied.*

If we take $\tilde{C} \cong \overline{1}$, then generalized soft metric space $(\tilde{X}, \mathcal{D})$ is a soft metric space.

Definition 3.4 *Let $(\tilde{X}, \mathcal{D})$ be a generalized metric space. Let $\{P_{\lambda,n}^x\}$ be a sequence in $\tilde{X}$ and $P_\lambda^x \tilde{\in} \tilde{X}$. We say that $\{P_{\lambda,n}^x\}$ is soft $\mathcal{D}$- converges to P_λ^x if $\{P_{\lambda,n}^x\} \tilde{\in} C(\mathcal{D}, \tilde{X}, P_\lambda^x)$.*

Example 3.5 *In Example 3.2, the sequence $\{P_{\lambda,n}^x\}$ is soft $\mathcal{D}$- converges to P_λ^0.*

Proposition 3.6 *Let $(\tilde{X}, \mathcal{D})$ be a generalized soft metric space. Let $\{P_{\lambda,n}^x\}$ be a sequence in $\tilde{X}$ and $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$. If $\{P_{\lambda,n}^x\}$ is soft $\mathcal{D}$- converges to P_λ^x and $\{P_{\lambda,n}^x\}$ is soft $\mathcal{D}$- converges to P_μ^y, then $P_\lambda^x \cong P_\mu^y$.*

Proof. Using the property (3), we have

$$\mathcal{D}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{C} \lim\sup_{n\to\infty} \mathcal{D}(P_{\lambda,n}^x, P_\mu^y) \cong \overline{0},$$

which implies from the property (1) that $P_\lambda^x \cong P_\mu^y$.

Definition 3.7 *Let $(\tilde{X}, \mathcal{D})$ be a generalized soft metric space. Let $\{P_{\lambda,n}^x\}$ be a sequence in $\tilde{X}$. We say that $\{P_{\lambda,n}^x\}$ is a soft $\mathcal{D}$- Cauchy sequence if $\lim_{m,n\to\infty} \mathcal{D}(P_{\lambda,n}^x, P_{\lambda,m+n}^x) \cong \overline{0}$.*

Example 3.8 *In Example 3.2, the sequence $\{P_{\lambda,n}^x\}$ is soft $\mathcal{D}$- Cauchy sequence.*

Definition 3.9 *A generalized soft metric space $(\tilde{X}, \mathcal{D})$ is said to be a soft $\mathcal{D}$-complete if every soft $\mathcal{D}$-Cauchy sequence in $\tilde{X}$ is converges to some soft element in $\tilde{X}$ and is said to be $\mathcal{D}$-incomplete if it is not $\mathcal{D}$-complete.*

Example 3.10 *A generalized soft metric space $(\tilde{X}, \mathcal{D})$ defined in Example 3.2 is soft $\mathcal{D}$-complete.*

If we take $(X', A) \tilde{\subset} \tilde{X}$ where $X'(\lambda) = (0,2]$ in real line and $Y(\mu) = \phi, \forall\, \mu \neq \lambda$, then there does not exist any $P_\lambda^{y'}$ in (X', A) such that $P_{\lambda,n}^x \to P_\lambda^{y'}$ in $(\widetilde{X'}, \mathcal{D})$.

$i.e.,$ $\{P_{\lambda,n}^x\}$ is a Cauchy sequence in $(\widetilde{X'}, \mathcal{D})$ that is not convergent in $(\widetilde{X'}, \mathcal{D})$. So, $(\widetilde{X'}, \mathcal{D})$ is not $\mathcal{D}$-complete.

Theorem 3.11 *Any soft b-metric on $\tilde{X}$ is a soft generalized metric on $\tilde{X}$.*

Proof. let d be a soft b-metric on $\tilde{X}$. We have just to proof that d satisfies the property (3). Let

$P_\lambda^x \tilde{\in} \tilde{X}$ and $\{P_{\lambda,n}^x\} \tilde{\in} C(d, \tilde{X}, P_\lambda^x)$. For every $P_\mu^y \tilde{\in} \tilde{X}$, we have,

$d(P_\lambda^x, P_\mu^y) \tilde{\leqq} \bar{s}\{d(P_\lambda^x, P_{\lambda,n}^x) + d(P_{\lambda,n}^x, P_\mu^y)\}$, for every natural number n.

Thus we have,

$$d\left(P_\lambda^x, P_\mu^y\right) \tilde{\leqq} \bar{s} \lim_{n\to\infty} \sup d(P_{\lambda,n}^x, P_\mu^y).$$

The property (3) is then satisfied with $\tilde{C} \cong \bar{s}$.

Definition 3.12 *Let $(\tilde{X}, \mathcal{D})$ be a generalized soft metric space and*

$f_\varphi: (\tilde{X}, \mathcal{D}) \to (\tilde{X}, \mathcal{D})$ be a soft mapping. Let $\tilde{k} \tilde{\in} \mathbb{R}(A)$ such that $\bar{0} \tilde{\lesssim} \tilde{k} \tilde{\lesssim} \bar{1}$. We say that f_φ is a soft $\tilde{k}$- contraction if

$\mathcal{D}(f_\varphi(P_\lambda^x), f_\varphi(P_\mu^y)) \tilde{\leqq} \tilde{k}\mathcal{D}(P_\lambda^x, P_\mu^y)$, for every $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$.

Example 3.13 *Consider a generalized soft metric space $(\tilde{X}, \mathcal{D})$ defined in Example 3.2. Define $f_\varphi: (\tilde{X}, \mathcal{D}) \to (\tilde{X}, \mathcal{D})$ by*

$f_\varphi(P_\lambda^x) \cong P_{\frac{\lambda}{2}}^{\frac{x}{2}}, \forall P_\lambda^x \tilde{\in} \tilde{X}$.

Now, $\forall P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$,

$$\mathcal{D}\left(P_{\frac{\lambda}{2}}^{\frac{x}{2}}, P_{\frac{\mu}{2}}^{\frac{y}{2}}\right) \cong \left|\frac{\bar{x}}{2} - \frac{\bar{y}}{2}\right| + \left|\frac{\bar{\lambda}}{2} - \frac{\bar{\mu}}{2}\right|$$

$$\cong \overline{\left(\frac{1}{2}\right)} \left[|\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}|\right]$$

$$\cong \overline{\left(\frac{1}{2}\right)} \mathcal{D}(P_\lambda^x, P_\mu^y)$$

Therefore, if we choose $\tilde{k} \cong \overline{\left(\frac{1}{2}\right)}$, then

$$\mathcal{D}\left(f_\varphi(P_\lambda^x), f_\varphi\left(P_\mu^y\right)\right) \tilde{\leqq} \tilde{k}\mathcal{D}\left(P_\lambda^x, P_\mu^y\right),$$

for every $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$.

$i.e.$, f_φ is a soft $\tilde{k}$- contraction mapping.

Proposition 3.14 *Let f_φ be a soft $\tilde{k}$-contraction mapping for some $\tilde{k} \tilde{\in} \mathbb{R}(A)$ with $\bar{0} \tilde{\lesssim} \tilde{k} \tilde{\lesssim} \bar{1}$. Then any fixed soft point $P_\omega^z \tilde{\in} \tilde{X}$ of f_φ satisfies*

$\mathcal{D}(P_\omega^z, P_\omega^z) \tilde{\lesssim} \infty \Rightarrow \mathcal{D}(P_\omega^z, P_\omega^z) \cong \bar{0}$.

Proof. Let $P_\omega^z \tilde{\in} \tilde{X}$ be a soft fixed point of f_φ such that $\mathcal{D}(P_\omega^z, P_\omega^z) \tilde{\lesssim} \infty$. Since f_φ is a soft $\tilde{k}$-contraction, We have,

$\mathcal{D}(P_\omega^z, P_\omega^z) \cong \mathcal{D}\left(f_\varphi(P_\omega^z), f_\varphi(P_\omega^z)\right) \tilde{\leqq} \tilde{k}\mathcal{D}(P_\omega^z, P_\omega^z)$,

which implies that $\mathcal{D}(P_\omega^z, P_\omega^z) \cong \bar{0}$, since $\bar{0} \tilde{\lesssim} \tilde{k} \tilde{\lesssim} \bar{1}$ and $\mathcal{D}(P_\omega^z, P_\omega^z) \tilde{\lesssim} \infty$.

Note: For every $P_\lambda^x \tilde{\in} \tilde{X}$, let

$\delta(\mathcal{D}, f_\varphi, P_\lambda^x) \cong sup\{\mathcal{D}(f_{\varphi^i}^i(P_\lambda^x), f_{\varphi^j}^j(P_\lambda^x)) : i, j \in \mathbb{N}\}$.

Theorem 3.15 *Suppose that the following conditions hold:*

- $(\tilde{X}, \mathcal{D})$ is complete;
- f_φ is a soft $\tilde{k}$- contraction where $\bar{0} \tilde{\lesssim} \tilde{k} \tilde{\lesssim} \bar{1}$;
- there exist $P_\lambda^x \tilde{\in} \tilde{X}$ such that $\delta(\mathcal{D}, f_\varphi, P_\lambda^x) \tilde{\lesssim} \infty$.

Then $\{f_{\varphi^n}^n(P_\lambda^x)\}$ soft $\mathcal{D}$- converges to P_ω^z, a fixed soft point of f_φ. Moreover, if $P_\mu^y \tilde{\in} \tilde{X}$ is another fixed soft point of f_φ such that $\mathcal{D}(P_\omega^z, P_\mu^y) \tilde{\lesssim} \infty$, then $P_\omega^z \cong P_\mu^y$.

Proof. Let $n \in \mathbb{N}$ $(n \geq 1)$. Since f_φ is a soft $\tilde{k}$-contraction, for all $i, j \in \mathbb{N}$, we have

$$\mathcal{D}\left(f_{\varphi^{n+i}}^{n+i}(P_\lambda^x), f_{\varphi^{n+j}}^{n+j}(P_\lambda^x)\right) \tilde{\leqq} \tilde{k}\, \mathcal{D}\left(f_{\varphi^{n-1+i}}^{n-1+i}(P_\lambda^x), f_{\varphi^{n-1+j}}^{n-1+j}(P_\lambda^x)\right),$$

which implies that,

$$\delta\left(\mathcal{D}, f_\varphi, f_{\varphi^n}^n(P_\lambda^x)\right) \tilde{\leqq} \tilde{k}\delta\left(\mathcal{D}, f_\varphi, f_{\varphi^{n-1}}^{n-1}(P_\lambda^x)\right).$$

Then for every $n \in \mathbb{N}$, we have

$$\delta\left(\mathcal{D}, f_\varphi, f_{\varphi^n}^n(P_\lambda^x)\right) \cong \tilde{k}^n \delta\big(\mathcal{D}, f_\varphi, P_\lambda^x\big)$$

Thus for every $n, m \in \mathbb{N}$, we have

$$\mathcal{D}\left(f_{\varphi^n}^n(P_\lambda^x), f_{\varphi^{n+m}}^{n+m}(P_\lambda^x)\right) \leqq \delta\left(\mathcal{D}, f_\varphi, f_{\varphi^n}^n(P_\lambda^x)\right) \leqq \tilde{k}^n \delta\big(\mathcal{D}, f_\varphi, P_\lambda^x\big).$$

Since $\delta(\mathcal{D}, f_\varphi, P_\lambda^x) \precsim \infty$ and $\overline{0} \precsim \tilde{k} \precsim \overline{1}$, we obtain

$$\lim_{n,m\to\infty} \mathcal{D}\left(f_{\varphi^n}^n(P_\lambda^x), f_{\varphi^{n+m}}^{n+m}(P_\lambda^x)\right) \cong \overline{0},$$

which implies that $\{f_{\varphi^n}^n(P_\lambda^x)\}$ is a soft $\mathcal{D}$- Cauchy sequence.

Since $(\tilde{X}, \mathcal{D})$ is soft $\mathcal{D}$- complete, there exists some $P_\omega^z \tilde{\in} \tilde{X}$ such that $\{f_{\varphi^n}^n(P_\lambda^x)\}$ is soft $\mathcal{D}$-convergent to P_ω^z.

On the other hand, since f_φ is a soft $\tilde{k}$-contraction, for all $n \in \mathbb{N}$, we have

$$\mathcal{D}\left(f_{\varphi^{n+1}}^{n+1}(P_\lambda^x), f_\varphi(P_\omega^z)\right) \leqq \tilde{k}\mathcal{D}\big(f_{\varphi^n}^n(P_\lambda^x), P_\omega^z\big).$$

Letting $n \to \infty$ in the above inequality, we get

$$\lim_{n\to\infty} \mathcal{D}\left(f_{\varphi^{n+1}}^{n+1}(P_\lambda^x), f_\varphi(P_\omega^z)\right) \cong \overline{0}.$$

Then $\{f_{\varphi^n}^n(P_\lambda^x)\}$ is a soft $\mathcal{D}$- convergent to $f_\varphi(P_\omega^z)$. By the uniqueness of limit, we get $P_\omega^z \cong f_\varphi(P_\omega^z)$, that is P_ω^z is fixed soft point of f_φ.

Now, suppose that $P_\mu^y \in \tilde{X}$ is a fixed soft point of f_φ such that $\mathcal{D}(P_\omega^z, P_\mu^y) \precsim \infty$. Since f_φ is a soft $\tilde{k}$-contraction, we have

$$\mathcal{D}\big(P_\omega^z, P_\mu^y\big) \cong \mathcal{D}\left(f_\varphi(P_\omega^z), f_\varphi\big(P_\mu^y\big)\right) \leqq \tilde{k}\mathcal{D}(P_\omega^z, P_\mu^y),$$

which implies that $P_\omega^z \cong P_\mu^y$.

4. Application

Choose $A = [-2, \infty)$ and $\tilde{X}(\lambda) = \left[-\frac{1}{4}, \frac{1}{4}\right], \forall\, \lambda \in A$.

Define $\mathcal{D}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A)^*$ by $\mathcal{D}(P_\lambda^x, P_\mu^y) \cong |\overline{x} - \overline{y}| + |\overline{\lambda} - \overline{\mu}|, \forall\, P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$.

Let $\{P_{\lambda,n}^x\}$, sequence of soft points of (X, A), such that $P_{\lambda,n}^x(\lambda) = \frac{1}{n}, \forall\, n \in \mathbb{N}$.

Then clearly $(\tilde{X}, \mathcal{D})$ is a generalized soft metric space. Also, $(\tilde{X}, \mathcal{D})$ is complete.

Define $\phi: A \to A$ by $\phi(\lambda) = \frac{\lambda}{2}, \forall\, \lambda \in A$ and $f: \left[-\frac{1}{4}, \frac{1}{4}\right] \to \left[-\frac{1}{4}, \frac{1}{4}\right]$ by $f(x) = \frac{x}{2}, \forall\, x \in \left[-\frac{1}{4}, \frac{1}{4}\right]$.

Let $f_\varphi: (\tilde{X}, \mathcal{D}) \to (\tilde{X}, \mathcal{D})$ *be such that* $f_\varphi(P_\lambda^x) \cong P_{\phi(\lambda)}^{f(x)}, \forall\, P_\lambda^x \tilde{\in} \tilde{X}$.

$$i.e., f_\varphi(P_\lambda^x) \cong P_{\frac{\lambda}{2}}^{\frac{x}{2}}, \forall\, P_\lambda^x \tilde{\in} \tilde{X}.$$

Then from Example 3.13, we have f_φ is a soft $\tilde{k}$-contraction.

Thus, all the conditions of Theorem 3.15 are satified. So, from the condition f_φ has a fixed soft point. Here P_0^0 is fixed soft point.

5. Conclusion

In the present paper, we have introduced the generalized soft metric space, and some of its basic properties are studied. Also, we have established some significant fixed point results in this setting. Our prospect is that this investigation has great weight and will support the researchers in cultivating new concepts in the field of soft fixed point results.

References

[1] A. Kharal, and B. Ahmad, Mappings on soft classes., New Math. Nat. Comput., 7 (3) (2011), 471-481.

[2] A. C. Guler, E. D. Yildirim, and O. B. Ozbakir, A fixed point theorem on soft G-metric spaces, J. Nonlinear Sci. Appl., **9** (2016), 885-894.

[3] A. C. Guler, and E. D. Yildirim, A note on soft *G*-metric spaces about fixed point theorems, Annals of Fuzzy Mathematics and Informatics, **12** (5) (2016), 691-701.

[4] B. R. Wadkar, R. Bhardwaj, V. N. Mishra, and B. Singh, Coupled soft fixed point theorems in soft metric and soft b-metric Space, Appl. Math. Inform. Mech., **9** (1) (2017), 59-73.

[5] B. R. Wadkar, B. Singh, and R. Bhardwaj, Coupled fixed point theorems with monotone property in soft b-metric space, Int. J. Math. Anal., **11** (8) (2017), 363-375.

[6] C. Vijender, A generalized metric space and related fixed point theorems, Int. Research J. of Eng. and Tech., **4** (7) (2017), 795-800.

[7] C. G. Aras, S. Bayramov, and V. Cafarli, A study on soft S-metric spaces, Commun. Math. Appl., **9** (4) (2018), 713–723.

[8] C. G. Aras, S. Bayramov, and V. Cafarli, Fixed point theorems on soft S-metric spaces. Commun. Math. Appl. **9** (4) (2018), 725–735.

[9] C. G. Aras, S. Bayramov, and M. I. Yazar, Soft D-Metric spaces, Bol. Soc. Paran. Mat., **38** (7) (2020), 137-147.

[10] D. Molodtsov, Soft Set Theory-First Results, Computers and Mathematics with Applications., **37** (1999), 19-31.

[11] P. K. Maji, R. Biswas, and A. R. Roy, Soft set theory, Comput. Math. Appl., **45** (2003), 555-562.

[12] P. Majumdar, and S.K. Samanta, On soft mappings, Comput. Math. Appl., **60** (2010), 2666-2672.

[13] S. Das, and S. K. Samanta, Soft Real Set. Soft Real Number And Their Properties, J. Fuzzy Math, **20** (3) (2012), 551-576.

[14] S. Das, and S. K. Samanta, Soft metric, Annals of Fuzzy Mathematics and Informatics, **6** (1) (2013), 77-94.

[15] S. Das, and S. K. Samanta, On soft metric spaces. J. Fuzzy Math., **21** (2013), 707–734.

[16] S. Das, P. Majumdar, and S.K., Samanta, On soft linear spaces and soft normed linear spaces, Ann. Fuzzy Math. Inform., **x** (x) (201y), 1-xx.

[17] Sk. Nazmul, and U. Badyakar, SOFT R-METRIC SPACES AND SOME OF ITS FIXED POINT RESULTS, Advances and Applications in Mathematical Sciences, **21** (6) (2022), 3431-3443.

[18] Sk. Nazmul, and U. Badyakar, SOFT S_b-METRIC SPACES AND SOME OF ITS PROPERTIES, Electronic Journal of Mathematical Analysis and Applications, **12** (1) (2024), 1-11.

[19] L. A. Zadeh, Fuzzy Sets, Inform. Control, **8** (1965), 103-112.

[20] M. Shabir, and M. Naz, On soft topological spaces., Comput. Math.

Appl., **61** (7) (2011), 1786–1799.

[21] M. Jleli, and B. samet, A generalized metric space and related fixed point theorems, Fixed Point Theory Appl, **61** (2015), 1-14.