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Analysis of Spontaneous Symmetry Breaking phenomenon in QFT

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Abstract

Symmetry plays a very vital role in physical system and it helps in understanding various phenomenon of nature. Beauty of symmetry lies not only in its original form but when it is broken (due to some external reasons) then also it opens doors to understand other related phenomenon. So, study of symmetry is like study of physics only. Concept of symmetry and its breaking helps in understanding very important phenomenon in physics like Superconductivity, phase transition in Ferromagnetism and mass generations of Bosons due to Higgs mechanism etc. The underlying mathematical model for symmetry analysis and its breaking is same in every phenomenon.

This report provides a detail explicit mathematical calculation of the different vacuum states (minima) of a Quantum Field Theoretical (QFT) model of 4D Lagrangian density, $\mathscr{L}$ of a complex scalar field (with a quartic interaction). These results are supported by the computational work which simulate the resulting potential depicting all points of minima. It is reported that the above system has degenerate vaccua as the locus of a ring in the complex plane for the mass parameter, m2< 0.This leads to the onset of Spontaneous Symmetry Breaking (SSB) in the potential term due to the breaking of gauge symmetry of the potential. This is a very interesting phenomenon because SSB plays an important role in Higgs Mechanism which is supposed to generate or attributes mass to all the particles (with mass) existing in the universe.

Keywords: Symmetry, QFT, Lagrangian density, quartic scalar fields, vacuum states, SSB

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1. Introduction

Physics associated with the study of symmetry is very rich, profound and interesting. It requires an elaborate discussion and presentation with the larger spectrum of audiences ranging from post graduate students to high end researchers. In physics, practically all great conservation laws of nature are due to existence of some sort of symmetry or invariance. As per Noether's theorem for every continuous symmetry there is a corresponding conserved current (energy, momentum, angular momentum etc.). Nobel laureate P. A Anderson in his famous article More is Different published in Science (1972) [1] quoted that "It is only slightly overstating the case to say that physics is the study of symmetry". This shows that why symmetry is so important aspect of nature and it demands a details study both theoretical as well as experimental. Nature loves symmetry and it is inherent in its body and soul. Governing laws of nature and theories not only valid for its existing symmetries but to the breaking of its symmetries also. This negative aspect (breaking)of symmetry is equally important in explaining various phenomenon of our universe. There are various mechanisms in nature wherein the symmetry can be hidden or broken. A more profound way of hiding symmetry is the phenomenon of spontaneous symmetry breaking. In this, laws of physics are symmetric but the ground (vacuum)state of the system does not respect the overall symmetry of the system.

Some examples of SSB from everyday life is suppose there is a spinning coin on a table and it can land either on its head or tail and it has no preferred choice for such landing. Both landing is equally probable (50-50 chances). As long the coin is rotating it has the symmetry or freedom of choice to land on any one side. Once it stopped spinning and choose to land on head or tail (minima or vacuum state) the initial symmetry is broken and system undergoes a SSB and initial symmetry of the system is no longer available to it.

Another, similar example is people sitting on a round dining table and they are symmetric (rotational) w.r.t the centre of the table. If any one person uses his left or right hand to pick up a spoon or plate then spontaneously the rotational symmetry is broken and system undergoes SSB as it chooses one particular state.

Phenomenon of SSB occurs in various fields like Material Science, High energy (particle) physics etc [2].Initially this concept was introduced by Landau to explain magnetic phase transition in Ferromagnets. In Ferromagnets all the atomic dipole moments are arranged either up or down and resulting in development of net magnetisation, M. When the temperature rises above the critical temperature $(T > T_c)$, Ferromagnets lose their net magnetisation and dipoles are randomly oriented owing to thermal energy and system becomes random and chaotic due to decrease in symmetry. The system acquired a radial symmetry (all directions are equally probable). The minima for this configuration occur at $M = 0$ only (single value).

When system is cooled down $(T < T_c)$, dipole moments start arranging themselves in up or down positions (both positions are equally probable) and a net magnetisation M is regained and system becomes ordered as initial symmetry is regained. M is acting as the order parameter for the given system. In the vicinity of T_c both the events occur. For $T < T_c$, system has minima at two different values of $M = +t, -t$ (t is symmetry breaking parameter for this system). $M = 0$ minimum state is no longer available in this case. Thus below T_c system falls spontaneously in one of the two minima values of the order parameter $M(\neq 0)$ available to them and thus breaking the radial symmetry of the original configuration. This loss or breaking of the symmetry leads to gain in the overall order of the system as value of the ordered parameter M increases.

Landau's path breaking concept of SSB is introduced to particle physics (Standard Model/ Electroweak theory) by Peter Higgs in 1960. He gives the concept of Higgs potential (H), Higgs Boson and postulated Higgs mechanism, which through SSB imparts masses to Higgs Bosons (W & Z). It is proposed that universe undergoes a SSB as it was cooling due to loss of energy during expansion. Just after the 10^{-11} s of the Big Bang explosion the universe undergoes SSB and slowly all the particles like electron, proton, neutron etc (initially they were massless) starts gaining their masses and formation of atoms takes place. Also, all the fundamental forces of nature (gravity, electromagnetism, strong and weak) got separate out from each other and loses their unified identity/ nature which they believed to have just before the onset of Big Bang. So SSB plays an important role in the formation and space time fabric of the universe in the present form as we see it today.

Since inception of the universe our mother nature playing and hiding with its symmetries and showing its different colours through its mistry particles (like God particles which is supposed to give masses to all the particles in the universe) and still it is holding many unsolved mysteries in its womb waiting to be revealed by the mankind in due course of time. Many new hidden symmetries may be discovered as the Grand Unification Theory (GUT) is still not taken its final shape and formulations. SSB theories may open many doors like here also. In order to verify the GUT theories, we have to create the situation in the lab with such a humongous amount of energy or temperature (10^{15} GeV) at which SSB was just set in (at least).

So, I choose this exciting phenomenon to work on it and write this article. This article will aid in the understanding the concept of SSB and its mathematical models (simulation etc.) in QFT (vacuum states and its derivations) and how the phenomenon of SSB occurs resulting in fascinating phenomenon like Higgs mechanism etc. This is particularly important for the university teachers &students of M.Sc. courses and beyond to understand above phenomenon in detail and lucid manner. Most of the available text books on Quantum Field Theory (QFT) [3] mention only about the ground states of a 4D complex scalar field (spin=0) with the quartic self-interaction terms for various values of its mass parameter (m). They do not show the explicit mathematical calculations through which different values of the ground states (minima) of such scalar fields are obtained. For a beginner, who just delved into the QFT field becomes quite difficult to understand or conceptualise the mathematical origin of these statements made in these standard reference books on QFT [4]. In this report a detailed mathematical calculation and simulations were done to give more insight in to the calculation of the ground states of a scalar complex field about its origin and physical interpretation through different types of plots (2D, 3D, ellipsoid of revolution etc). Such detailed mathematical approach to this problem is not reported yet.

2. Mathematical Preliminaries and methodology

In the free field theory, simplest form of Lagrangian density, $\mathcal{L}_0$ for a complex scalar field ($\emptyset$) and $\emptyset^*$ is complex conjugate of $\emptyset$ with m as the mass (parameter) of the field, is given by

$$\mathcal{L}_0 = (\partial_\mu \emptyset)(\partial^\mu \emptyset^*) - m^2(\emptyset^* \emptyset) \dots\dots\dots....(1)$$

If we use this simple form of Lagrangian then we will obtain an equation of motion for scalar field $\emptyset$. This classical field also satisfies, Klein-Gordon (KG) equation. In this field, particles (zero spin) can only be excited or de-excited to given available energy states (satisfying certain underlying conditions) but they do not interact among themselves. Thus, such free field can only give emission or absorption spectra. Here number of particles involved remains conserved I,e creation or annihilation of particles is not allowed.

If we add or include higher order terms like$\mathcal{L}_{int.}$in this free field Lagrangian and it can be treated as a weak perturbation to the given system (assuming valid at all energy regime). Then such perturbed Lagrangian results in the crop of very interesting phenomenon called interaction among the field particles themselves and it is known as self-interaction.

Thus, new Lagrange is now, $\mathcal{L}_{new} = \mathcal{L}_0 + \mathcal{L}_{int.}$, Where $\mathcal{L}_{int.}$ is the interaction Lagrangian density term (perturbation).

If we are working in a low energy interaction regime, then we can consider $\mathcal{L}_{int.}$ as quartic interaction which is type of self-interaction in a scalar field. Quartic interaction term is represented by $\mathcal{L}_{int.} = -\lambda(\emptyset^*\emptyset)^2$, derived from weakly coupled $\emptyset^4$ theory [5]. This term induces interaction among the field particles and gives rise to many interesting phenomena like particle creation or annihilation, which means that total number of particles does not remain conserved unlike in earlier case. λ (>0) is known as the coupling constant for the given interaction.

Thus, we can write new Lagrangian density, eqn. (1) as below

$$\mathcal{L}_{new} = \mathcal{L} = (\partial_\mu\emptyset)(\partial^\mu\emptyset^*) - m^2(\emptyset^*\emptyset) - \lambda(\emptyset^*\emptyset)^2 = (\partial_\mu\emptyset)(\partial^\mu\emptyset^*) - V(\emptyset,\emptyset^*) \qquad (2)$$

This can be treated as an effective quantum field theory (EQFT) in low energy interactions.

Total potential is now, $V(\emptyset,\emptyset^*) = m^2(\emptyset^*\emptyset) + \lambda(\emptyset^*\emptyset)^2 = m^2|\emptyset|^2 + \lambda|\emptyset|^4$

Here mass term is very important, later, we will see that when $m^2 < 0$, the last quartic (interaction) term is responsible for spontaneous symmetry breaking of the potential $V(\emptyset,\emptyset^*)$ so it is also known as the symmetric breaking parameter. This parameter decides whether the given potential term will undergo spontaneous breaking or not.

Now, the complex field $\emptyset$ can be expressed as a combination of real and imaginary parts as follows,

$$\emptyset = \frac{1}{\sqrt{2}}(\emptyset_1 + i\emptyset_2, \emptyset^* = \frac{1}{\sqrt{2}}(\emptyset_1 - i\emptyset_2) \qquad (3)$$

If we put eqn. (3) in eqn. (2), we obtain after simplification that

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\emptyset_1)(\partial^\mu\emptyset_1) + \frac{1}{2}(\partial_\mu\emptyset_2)(\partial^\mu\emptyset_2) - \frac{m^2}{2}(\emptyset_1^2 + \emptyset_2^2) - \frac{\lambda}{4}(\emptyset_1^2 + \emptyset_2^2)^2 \qquad (4)$$

The potential function $V(\emptyset,\emptyset^*)$, from the above is now reduced to

$$V(\emptyset_1,\emptyset_2) = \frac{m^2}{2}(\emptyset_1^2 + \emptyset_2^2) + \frac{\lambda}{4}(\emptyset_1^2 + \emptyset_2^2)^2 \qquad (5)$$

The above potential now depends upon the values of two scalar fields $\emptyset_1 and\ \emptyset_2$.

Now our aim is to find the vacuum state or ground state energy of the given potential $V(\emptyset,\emptyset^*)$. This requires to find the extrema of the above potential. As per calculus, for maxima, the second derivative should be negative at the point where first derivative is zero. First derivatives are calculated as below

$$\frac{\partial V}{\partial\emptyset_1} = m^2\emptyset_1 + \lambda\emptyset_1(\emptyset_1^2 + \emptyset_2^2), \qquad \frac{\partial V}{\partial\emptyset_2} = m^2\emptyset_2 + \lambda\emptyset_2(\emptyset_1^2 + \emptyset_2^2)$$

Their corresponding second derivatives will be calculated as

$$\frac{\partial^2 V}{\partial \emptyset_1{}^2} = m^2 + \lambda(3\phi_1^2 + \emptyset_2^2) = r(\text{say}) \qquad (6)$$

$$\frac{\partial^2 V}{\partial \emptyset_2{}^2} = m^2 + \lambda(3\phi_2^2 + \emptyset_1^2) = t \qquad (7)$$

$$\frac{\partial^2 V}{\partial \emptyset_1 \partial \emptyset_2} = 2\lambda \emptyset_1 \emptyset_2 = s \qquad (8)$$

For two variables $\emptyset_1 and\, \emptyset_2$, we have to determine the sign of the quantity

$(rt - s^2)$. This will decide the corresponding extremum values.

$$\text{setting } \frac{\partial V}{\partial \emptyset_1} = 0 \text{ and } \frac{\partial V}{\partial \emptyset_2} = 0$$

This will in turn gives,

$$\emptyset_1[m^2 + \lambda(\emptyset_1^2 + \emptyset_2^2)] = 0 \qquad (9)$$

$$\emptyset_2[m^2 + \lambda(\emptyset_1^2 + \emptyset_2^2)] = 0 \qquad (10)$$

From eqn (9) and (10) we can conclude possible solutions are

$\emptyset_1 \& \emptyset_2 = 0$ (local minima) & $[m^2 + \lambda(\emptyset_1^2 + \emptyset_2^2)] = 0$ if we assume $\emptyset_1^2 + \emptyset_2^2 = \mu^2\ (say)$

then$[m^2 + \lambda\mu^2] = 0$ or$\mu^2 = {-m^2}/_{\lambda}$

Now,$m^2 < 0$ or$m^2 > 0$ so two different cases arises and all these possible cases are further explained under different sub categories as below.

Case 1: $m^2 > 0$,

In this case bracketed terms in eqn. (9), (10) will always be positive non zero term I,e

$m^2 + \lambda(\emptyset_1^2 + \emptyset_2^2) \neq 0$ **.**

Then solution from eqn. (9), (10) will be given by

$\Rightarrow \emptyset_1 = 0 \,\&\, \emptyset_2 = 0$

Thus, in the complex plane, $O(0,0)$ is an extremum point. At this extremum point we have to calculate the value of the quantity $(rt - s^2)$ as follows

$(rt - s^2) = [m^2 + \lambda(3\phi_1^2 + \emptyset_2^2)]\ [m^2 + \lambda(3\phi_2^2 + \emptyset_1^2)]$ - $4\lambda^2\emptyset_1^2\emptyset_2^2$

$$= m^4 + other\ terms > 0. \qquad (11)$$

Thus, the value of $(rt - s^2)$ is positive so the sign of r will decide whether the point $O(\emptyset_1, \emptyset_2)$ is maxima or minima.

If $r > 0$, $(\emptyset_1, \emptyset_2)$ is minima

$r < 0$, $(\emptyset_1, \emptyset_2)$ is maxima

It can be explicitly checked that

$$r = m^2 + \lambda(3\phi_1^2 + \emptyset_2^2) = m^2 + (other\ terms) > 0.$$

Thus, the point $O(0,0)$ in complex plane of $\emptyset_1 and\, \emptyset_2$ is point of minima as shown in fig. (1).

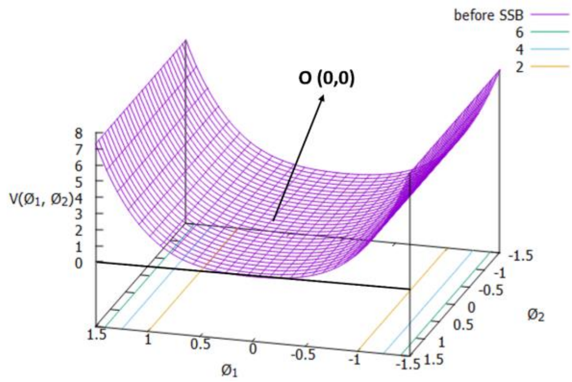

Fig. 1. 2D plot of the potential $V(\emptyset_1, \emptyset_2)$ with origin at $O(0,0)$ for $m^2 > 0$

Potential $V(\emptyset_1, \emptyset_2)$ at point $O(0,0)$ is symmetric so the corresponding ground state lying at the origin is also symmetric under global ($\theta = constant$) continuous $\cup(1)$ gauge transformations as below

$$\emptyset(x) \rightarrow \emptyset'(x) = \emptyset(x)e^{-i\theta} \qquad (12)$$

$$\emptyset^*(x) \rightarrow (\emptyset^*)'(x) = \emptyset^*(x)e^{i\theta} \qquad (13)$$

Which happens to be a continuous symmetry of the starting Lagrangian density of eqn. (2). As a consequence, ground state is also found to satisfy the above continuous $\cup(1)$ gauge symmetry. Thus, we conclude that there is no SSB for the case $m^2 > 0$.

Case 2: $m^2 < 0$,

This is the most important case among all and it leads to many interesting & complicated phenomena. Case 2 has been further sub divide into sub cases (2.1 &2.2), depending on the possible set (two) of solutions to the eqns. (9) & (10).

Case 2.1: $\emptyset_1 = 0$, $\emptyset_2 = 0$. This corresponds to the point $O(0,0)$, origin of the complex plane. At this point following conditions are satisfied, namely

$$rt - s^2 = m^4 > 0 \qquad (14)$$

$$\& \, r = m^2 < 0$$

This shows that, unlike the previous case, here $r < 0$ (negative). Thus, the point $O(0,0)$ now here corresponds to the point of maxima instead of minima as we have in case 1. Thus, the origin is lifted up (excited) from the known groundstate and it no longer corresponds to the ground state (vacuum) thus breaks the ground state symmetry it enjoyed in the previous case. This result is graphically shown in fig. (2)

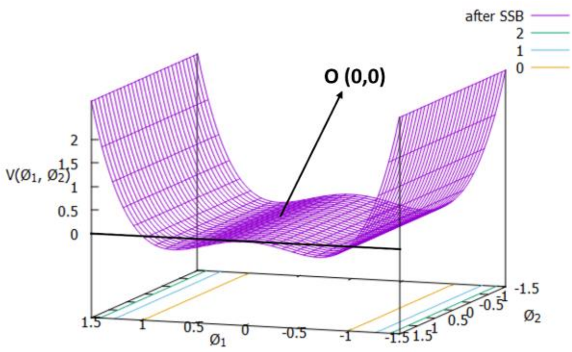

Fig. 2. 2D plot of $V(\emptyset_1, \emptyset_2)$ for $m^2 < 0$.

Case 2.2. $\emptyset_1 = \widetilde{\emptyset_1}, \emptyset_2 = \widetilde{\emptyset_2}$(set of probable solution)

This reduces the original eqns. (9), (10) to the following eqns. (15), (16) respectively as.

$$[m^2 + \lambda(\emptyset_1^2 + \emptyset_2^2)] = 0 \qquad (15)$$

$$[m^2 + \lambda(\emptyset_1^2 + \emptyset_2^2)] = 0 \qquad (16)$$

When we solve the eqn. (15) & (16) we will get infinite number of solutions for the pair $\emptyset_1 and\, \emptyset_2$. Now we will have to dig out the possible ground states from this infinite number of probable solutions. We have to use Taylor Series to find the extrema values as we have done from basic mathematics (partial derivatives with two variables) [**6**]. Any function, $f(x,y)$ will have maxima or minima at point $x = a, y = b$ according to condition,

$$f(a+h, b+k) < f(a,b) \text{for maxima}$$
$$> f(a,b) \text{for minima} \qquad (17)$$

Alternately, if $\Delta f = [f(a+h, b+k) - f(p,q)]$ is of the same sign for all small values of h, k and if the sign is negative, then $f(a,b)$ is maximum and it is minimum when the sign is positive. Infinitesimal variations are $\delta x = h, \delta y = k$.

Now we will apply this result to potential function $V(\emptyset_1, \emptyset_2)$ **t**o calculate its different maxima or minima around a certain given solution.

Let us assume $\widetilde{\emptyset_1}$ & $\widetilde{\emptyset_2}$ are the solutions of eqn. (15) or (16).

If $\delta\widetilde{\emptyset_1}(=h)$ & $\delta\widetilde{\emptyset_2}(=k)$ are small displacements around the point P $(\widetilde{\emptyset_1},\widetilde{\emptyset_2})$

Then we can have $\emptyset_1=\widetilde{\emptyset_1}$ & $\emptyset_2=\widetilde{\emptyset_2}$ in eqn. (15) and (16).

Taylor expansion of the potential $V(\emptyset_1,\emptyset_2)$ around this solution $(\widetilde{\emptyset_1},\widetilde{\emptyset_2})$ can be explicitly expanded as

$$V(\widetilde{\emptyset_1}+\delta\widetilde{\emptyset_1},\widetilde{\emptyset_2}+\delta\widetilde{\emptyset_2})=V(\widetilde{\emptyset_1},\widetilde{\emptyset_2})+\left[\frac{\partial V}{\partial\emptyset_1}|_{\emptyset_1=\widetilde{\emptyset_1}}\delta\emptyset_1+\frac{\partial V}{\partial\emptyset_2}|_{\emptyset_2=\widetilde{\emptyset_2}}\delta\emptyset_2\right]$$

$$+\frac{1}{2}\left[\frac{\partial^2 V}{\partial{\emptyset_1}^2}|_{\emptyset_1=\widetilde{\emptyset_1}}(\delta\emptyset_1)^2+\frac{\partial^2 V}{\partial{\emptyset_2}^2}|_{\emptyset_2=\widetilde{\emptyset_2}}(\delta\emptyset_2)^2+2\frac{\partial^2 V}{\partial\emptyset_1\partial\emptyset_2}|_{\widetilde{\emptyset_1},\widetilde{\emptyset_2}}(\delta\emptyset_1)(\delta\emptyset_2)\right]$$

$$\Rightarrow\Delta V=0+h^2\frac{\partial^2 V}{\partial\emptyset_1^2}|_{\emptyset_1=\widetilde{\emptyset_1}}+2hk\frac{\partial^2 V}{\partial\emptyset_1\partial\emptyset_2}|_{\widetilde{\emptyset_1},\widetilde{\emptyset_2}}+k^2\frac{\partial^2 V}{\partial{\emptyset_2}^2}|_{\emptyset_2=\widetilde{\emptyset_2}}$$

$$=h^2\{m^2+\lambda(3\widetilde{\emptyset_1^2}+\widetilde{\emptyset_2^2})\}+2hk(2\lambda\widetilde{\emptyset_1}\widetilde{\emptyset_2})+k^2\{m^2+\lambda(\widetilde{\emptyset_1^2}+3\widetilde{\emptyset_2^2})\}$$

$$=\lambda(h\widetilde{\emptyset_1}+k\widetilde{\emptyset_2})^2>0.$$

$\Rightarrow \Delta V>0.$

$\Rightarrow(\widetilde{\emptyset_1},\widetilde{\emptyset_2})$ is the solution corresponding to minimum.

This minimum is described by the following equation

$$\emptyset_1^2+\emptyset_2^2=\widetilde{\emptyset_1^2}+\widetilde{\emptyset_2^2}=\mu^2, where\ \mu=\sqrt{\frac{-m^2}{\lambda}} \quad (18)$$

Where, μ represents the radius of the circle/ ring made by the locus of points of which corresponds to all minima in the complex plane at a distance μ from the centre of the complex plane. It is also known as the symmetry breaking parameter for the given potential V.

Thus, for the case $m^2<0$ the potential is still symmetric but the minimum value of energy no longer corresponds to unique value of $\emptyset_1$&$\emptyset_2$ but instead we obtain a degenerate vacuum (I,e the minima as a ring in the complex plane defined by $\emptyset_1^2+\emptyset_2^2=\mu^2$)as shown in fig. (2).

Thus, in sub case **2.1**, $(m^2>0)$ origin $O(0,0)$ is the ground state with symmetry but in sub case **2.2**$(m^2<0)$ the same origin now becomes excited state and generating a new degenerate ground state giving rise to the phenomenon of SSB [7].

3. Simulation

Generated data points $(\emptyset_1,\emptyset_2)$are used in calculation of the corresponding values of the potential V$((\emptyset_1,\emptyset_2))$ and then it is plotted in gnuplot interface [**8**]. Using this platform two plots are generated corresponding to the cases $m^2>0$ (before SSB) as shown in fig. (3) and $m^2<0$ (after SSB)as plotted in fig.(4).

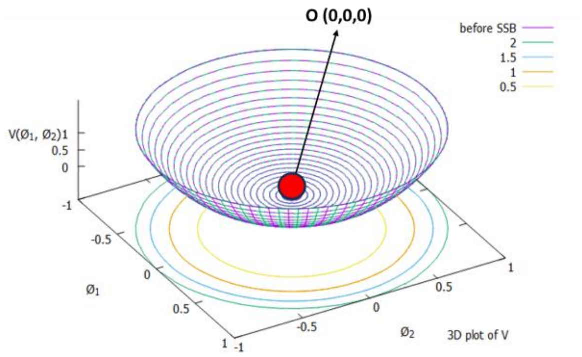

Fig. 3. 3D plot of $V(\emptyset_1,\emptyset_2)$with (0,0,0) as the base (minima) for$m^2>0$.

Fig. (3) is actually a paraboloid of revolution about the origin O (0,0,0). It clearly shows the symmetric nature of the plot about O. Here O is the global minima for the given potential. When a particle is placed at O it cannot go anywhere down (local minima not available) and it always enjoys the rotational symmetry of the given potential. Fig. (4) shows an interesting symmetry pattern in which the origin O in now lifted above Z=0 plane and now it is no longer the usual global minimum position and instead a loci (ring of infinite degenerate states of radius μ) of minima is created just below Z= 0 plane. This minimum is now becomes a global minimum. When a particle chooses one of these minima (degenerate) position spontaneously, the U (1) gauge symmetry is broken and it is known as the phenomenon of *Spontaneous Symmetry Breaking* (SSB). O now becomes unstable maxima for the given field.

The geometrical shape of this potential, upon SSB, becomes similar to a "Mexican cow boy hat"and this type of Potential function is also known as "*Sombrero Potential*" or "*Higgs Potential*".

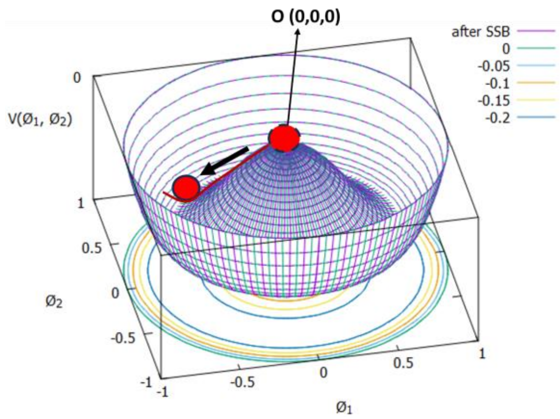

Fig. 4..3D plot of $V(\emptyset_1, \emptyset_2)$ with (0,0,0) as the base (maxima) for$m^2 < 0$.

When a particle is placed at O (as shown in Fig. 4.) it can roll down and fall in any available degenerate minima (local) and it causes SSB of the potential V. Due to this preferential behaviour of the particle the rotational symmetry is no longer available to it which it enjoyed earlier when placed at the global maxima O. So, the system remains always symmetric but the ground state ceases or break it. SSB phenomena occurs when the equation of motion (Lagrangian) satisfies certain symmetries but its ground state energy (vacuum) solutions cease to respect that symmetry, spontaneously under certain conditions. When the system goes either left or right-side lobe of the vacuum solution point $O(0,0)$of the potential as in fig. (2), the symmetry is instantly broken, but entire symmetry of the parent Lagrangian remains intact. System spontaneously transforms from symmetric to asymmetric one.

4. Conclusion

From the 3D plot in fig. (4) it is clear that when the system undergoes SSB, it has an infinite set of vacuum states, each corresponding to a point on the circumference of a circle of radius μ. Any point on the ring of minima is equivalent, as they can be obtained from any point on the circumference, by applying the condition of gauge transformation as given in eqn. (12), (13). Different ground states are orthogonal to one another. When an addition interaction terms (quartic, $\emptyset^4$ type) is added to free field potential like a small perturbation about the minima, it spontaneously causes symmetry to break in the given potential when $m^2 < 0$. This gives rise to degenerate symmetric ground states to the potential over a ring (*Mexican hat*). In this, symmetry is broken but actually it is hidden. Thus, from the above mathematical analysis, we have found several extrema values of a given potential and established whether those extrema correspond to maxima or minima. Also studied the behaviour of potential around those inflection points w.r.t various symmetries.

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